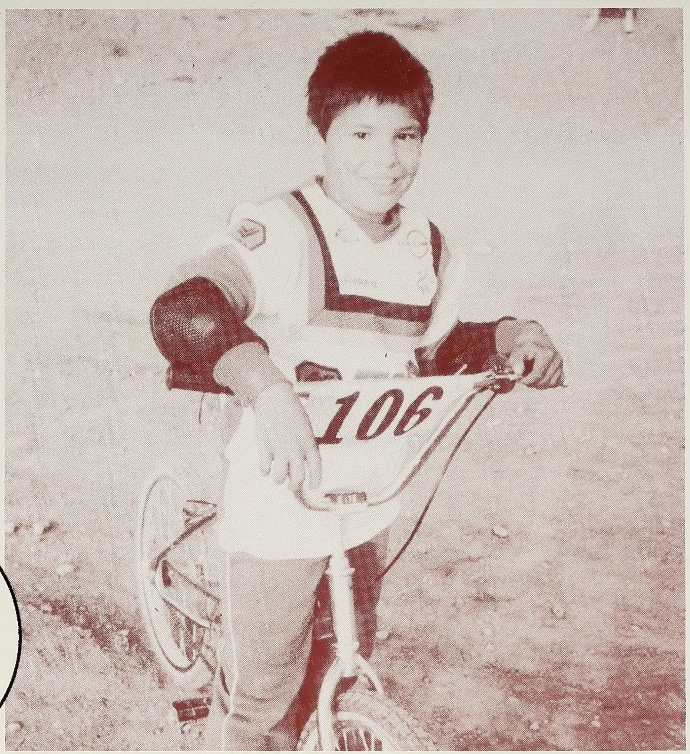
WHOLE NUMBERS
and INTEGERS

MODULE 2



MATHEMATICS 7

Distance Learning

Alberta
EDUCATION

Mathematics 7

Module 2: Whole Numbers and Integers

MODULE BOOKLET

Mathematics 7
Student Module
Module 2
Whole Numbers and Integers
Alberta Distance Learning Centre
ISBN No. 0-7741-0120-2

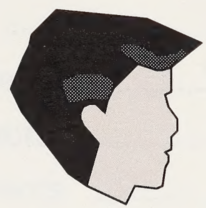
Cover Photo: WESTFILE INC.

Welcome to Module 2!

We hope you'll enjoy your study of ***Whole Numbers and Integers*** *.*

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,



turn on your audiocassette and listen.

CONTENTS AT A GLANCE

Module Introduction 1

Part One 5

Section 1: Getting Set 7

Section 2: Reading and Writing Whole Numbers 17

Section 3: Comparing and Ordering Whole Numbers 27

Section 4: Rounding Whole Numbers 35

Section 5: Estimating Sums 43

Section 6: Estimating Differences 55

Section 7: Estimating Products 61

Section 8: Estimating Quotients 71

Section 9: Finding Sums 79

Section 10: Finding Differences 87

Section 11: Finding Products 97

Section 12: Finding Quotients 107

Section 13: Summary 121

Part Two 123

Section 14: Getting Set 125

Section 15: Adding Mentally 129

Section 16: Subtracting Mentally 143
Section 17: Multiplying Mentally 151
Section 18: Dividing Mentally 165
Section 19: Order of Operations 177
Section 20: Summary 191

Part Three 193

Section 21: Getting Set 195
Section 22: Multiples 203
Section 23: Factors 211
Section 24: Prime and Composite Numbers 221
Section 25: Prime Factors 233
Section 26: Divisibility 243
Section 27: Powers 253
Section 28: Recognizing Integers 267
Section 29: Adding Integers 279
Section 30: Summary 295

Module Conclusion 297

Appendix 301

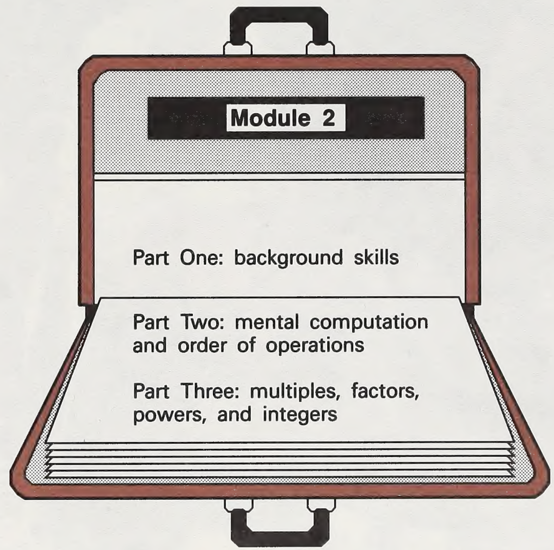
MODULE INTRODUCTION

What Lies Ahead

This Module Introduction will give you an overview of Module 2.

Working Together

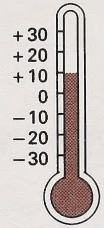
In Module 2 you will be working with whole numbers and integers. There are three parts to the module. This is how the module is organized.



Whole numbers and integers are used in daily life. You are probably very familiar with whole numbers. They are often used to represent people, places and products.

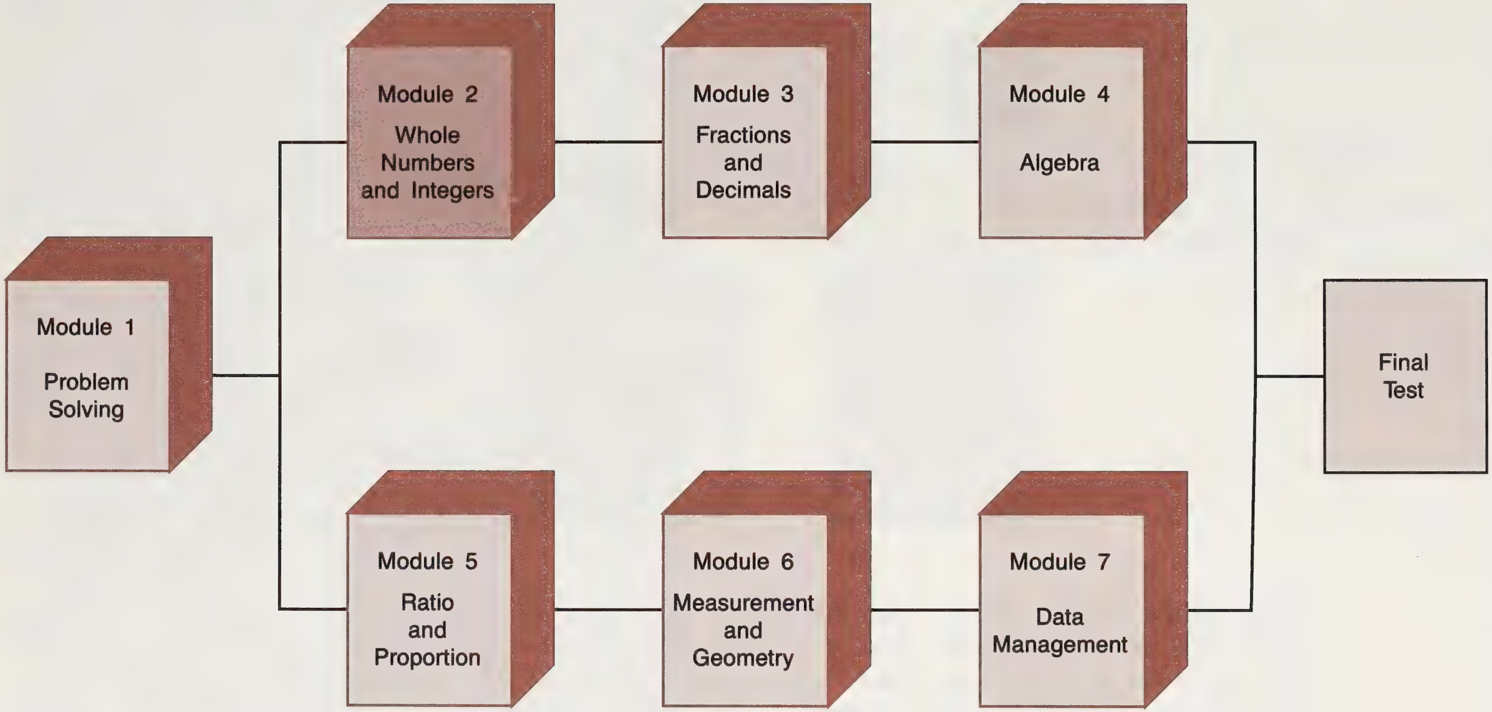
WESTFILE, INC.

You are probably less familiar with integers. One situation where you have used integers is measuring temperature.



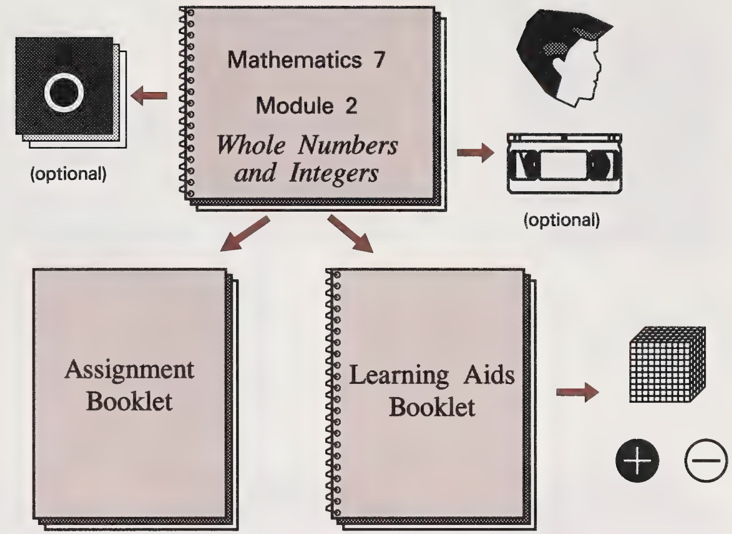
Another situation where you may have used integers is measuring height above sea level or below sea level.

Course Overview



Mathematics 7 has seven modules and a final supervised test. This module booklet is part of Module 2.

Module 2 Components

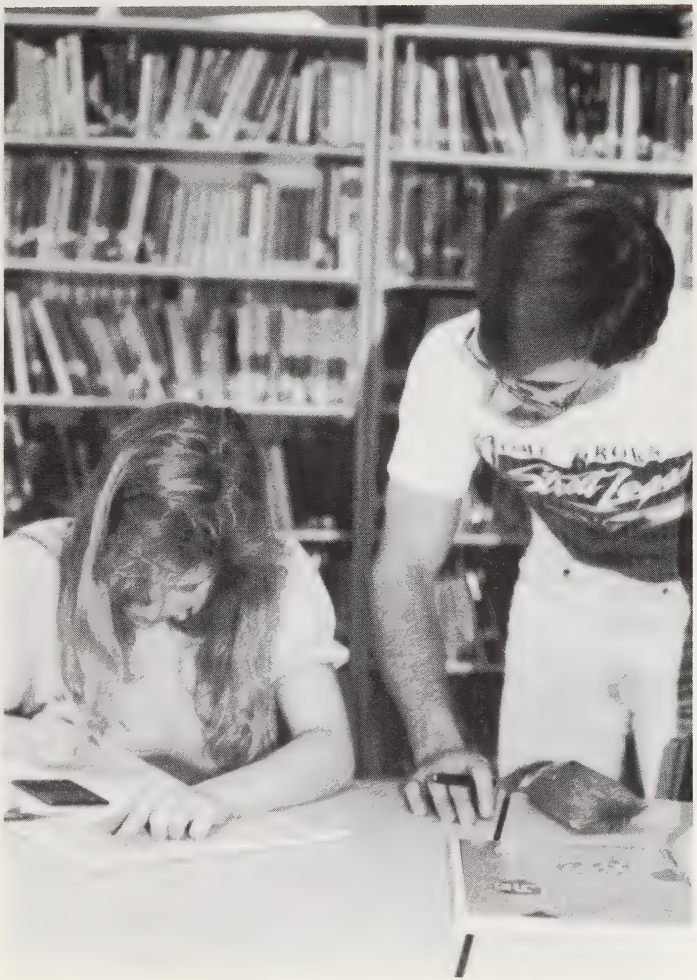


This module booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional; there are print alternatives. You should see your learning facilitator to check your answers to the activities in this booklet.

This module booklet is not to be submitted for a grade. **Your mark on this module will be determined by your work in the assignment booklet.**

Take time to preview the module booklet before beginning Section 1.

PART ONE



Sections 1-13 deal with whole number skills previously developed in earlier schooling.

It is important that you maintain these skills with whole numbers.

Reading and writing whole numbers, comparing, ordering and rounding whole numbers, and performing operations on whole numbers are skills which you will use throughout your life.

GETTING SET

What Lies Ahead

This section will test these skills.

- reading and writing whole numbers
- comparing and ordering whole numbers
- rounding whole numbers
- estimating whole number sums, differences, products, and quotients
- finding exact whole number sums, differences, products, and quotients using paper and pencil, and using a calculator
- checking the reasonableness and accuracy of whole number sums, differences, products, and quotients

Working Together

The pretest in this section will help you and your learning facilitator discover your strengths and weaknesses.

Pretest

Space for Your Work

1. Express in expanded form.

   a. 2 984

   b. 30 278

   c. 2 360 585

2. Express in standard form.

   a. $8\,000 + 300 + 40 + 7$

   b. $(1 \times 100\,000) + (3 \times 10\,000) + (8 \times 100) + (2 \times 1)$

   c. five hundred ninety

   d. three million seventy-six thousand two hundred eighty-eight

Space for Your Work

3. Write in words.

   a. 1 024

   b. 13 545 600

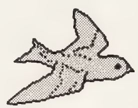
4. Order from greatest to least.

   a. 19, 911, 91, 191, 90

   b. 1234, 3421, 3241, 4321, 1423

5. Tell whether the numbers are exact or rounded.

   a. The bird survey recorded over 1800 warblers.



   b. Edmonton has about 600 000 people.

   c. August has thirty-one days.

Space for Your Work

6. Complete.

		Rounded to the nearest		
		ten	hundred	thousand
a.	879			
b.	8 528			
c.	94 846			

7. Estimate.

a.
$$\begin{array}{r} 64 \\ +\ 29 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 52\,876 \\ +\ \ 8\,093 \\ \hline \end{array}$$

c. 94 − 37

d.
$$\begin{array}{r} 187\,201 \\ -\ \ 88\,193 \\ \hline \end{array}$$

Space for Your Work

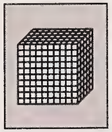
e. $$\begin{array}{r} 28 \\ \times\ 19 \\ \hline \end{array}$$

f. 307×68

g. $7\overline{)33\,310}$

h. $4233 \div 63$

Space for Your Work



8. Model the following using base 10 blocks.

   a. $65 + 38$

   b. $94 - 57$

   c. 127×6

   d. $7\overline{)343}$

Space for Your Work

9. Compute the following. Do **not** use a calculator.

a. $65 + 38$

b. $\begin{array}{r} 53\,786 \\ +\quad 8\,094 \\ \hline \end{array}$

c. $94 - 57$

d. $\begin{array}{r} 188\,203 \\ -\quad 77\,192 \\ \hline \end{array}$

e. 27×6

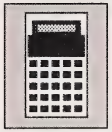
f. $\begin{array}{r} 586 \\ \times\quad 32 \\ \hline \end{array}$

g. $7\overline{)343}$

h. $4395 \div 63$

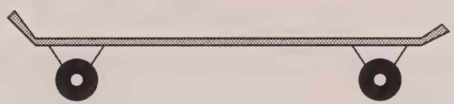
Space for Your Work

Use a calculator in Questions 10-12.



10. a. What is the total cost of the skateboard?

    b. How much change is left from $200?



Deck $88	Trucks $34
Wheels $42	Bearings and tape.... $ 8

11. Gloria's heart beats an average of 69 times in one minute. How many times does it beat in one hour?

12. The Petersons travelled 570 km in 6 hours on the first day of their vacation. Find the distance travelled in one hour.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

READING AND WRITING WHOLE NUMBERS

What Lies Ahead

In this section you will review these skills.

- identifying place value in whole numbers
- writing whole numbers in expanded form
- reading and writing whole numbers in standard form
- reading and writing whole numbers in words

In this section you will use these words.

- digit
- numeral
- place value
- standard form
- expanded form
- word form

Working Together

You have already learned a great deal about place value in earlier schooling. This section reinforces your ability to identify place value, and to read and write whole numbers.

Learning Aids Activities

You will begin by investigating place value using concrete models of numbers. you will need base 10 blocks for the Learning Aids Activities.

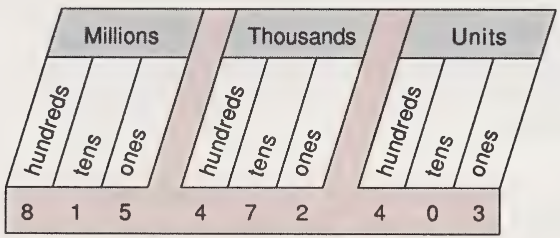
Turn to Exercise A in the *Learning Aids Booklet* and do the activities for this section.

Working Together

Now that you have examined concrete models of numbers, you should have a better understanding of place value.

Place Value

A place-value chart helps you to understand the value of digits in a number.



The 5 in 815 472 403 has a value of 5 millions

or $5 \times 1\,000\,000$.

The 7 has a value of 7 ten thousands

or $7 \times 10\,000$.

The 0 has a value of 0 tens

or 0×10.

Note

- Digits are in groups of three.
- Each group of three is called a **period.**
- Each period has a ones, tens and hundreds place.

Expanded Form

Numbers are sometimes written in expanded form. Expanded form is the sum of each digit and its place value.

The expanded form of 815 472 403 is

$$(8 \times 100\,000\,000) + (1 \times 10\,000\,000) +$$
$$(5 \times 1\,000\,000) + (4 \times 100\,000) + (7 \times 10\,000) +$$
$$(2 \times 1\,000) + (4 \times 100) + (0 \times 10) + (3 \times 1)$$

Note

Zero digits may be omitted in expanded form.

Here's a way to remember what expanded form and standard form mean.

Think of an accordian. Expanded form is like the accordian when it is spread out.

Standard form is like the accordian when it is closed.

Standard Form

Numbers are usually written in standard form.

This jet has travelled 370 845 291 km.

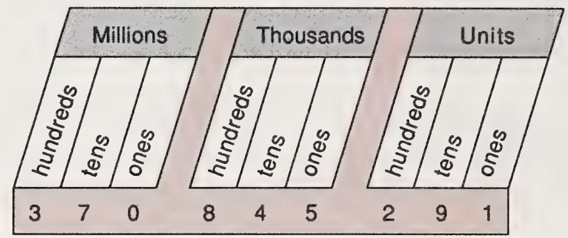
370 845 291 is the standard form of the number.

Note

- You will sometimes see commas used to separate the different periods. In the metric system commas are **not** written; spaces are used to separate the periods.
- The space in a 4-digit numeral is optional; a space **must** be used in numerals with 5 more digits.
- Zeros must be written in standard form to hold the place value.

How would you read 370 845 291 in words?

A place-value chart helps you to read large numbers.



370 845 291 is read "370 million 845 thousand 291."

Note

- The word *and* is not used in reading whole numbers.
- Zeros are not read.

Practice Activities

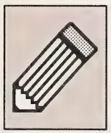
Space for Your Work

Computer Alternative

1. Do Lessons 1, 3, 4 of the disk "Numbers and Numeration" from the package, *Computer Drill and Instruction: Mathematics*, Level D (SRA).

   Read the instructions in the folder with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the ? key.

Print Alternative



2. Express in standard form.

   a. $60\,000 + 8000 + 500 + 30 + 4$

   b. $900\,000 + 50\,000 + 6000 + 300 + 2$

   c. $(5 \times 100\,000) + (3 \times 10\,000) + (7 \times 1000) + (9 \times 100) + (2 \times 10)$

   d. $(9 \times 1\,000\,000) + (8 \times 100\,000) + (5 \times 10\,000) + (4 \times 1000)$

   e. $(7 \times 100\,000) + (6 \times 10\,000) + (3 \times 100) + (6 \times 1)$

Space for Your Work

3. Express in expanded form.

   a. 98 100

   b. 50 543

   c. 73 725

   d. 3 140 000

   e. 80 500 000

4. Write in word form.

   a. 98 100

   b. 6853

   c. 7062

Space for Your Work

5. Write in standard form.

   a. seventy-five thousand one

   b. six thousand seventy-five

   c. one million eight hundred five

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Do you like big numbers? The chart below shows the names of the periods of a very large number.

decillions	nonillions	octillions	septillions	sextillions	quintillions	quadrillions	trillions	billions	millions	thousands	units
123	456	789	012	345	678	901	234	567	890	123	456

Do some research to answer the following.

1. Tell or guess where the names of the periods came from?

Space for Your Work

2. Is a billion the same amount in England as in Canada?

3. Name a situation where a large number such as decillions might be needed.

4. What does *a zillion* mean?

See your learning facilitator to check your answers and to receive further instructions.

COMPARING AND ORDERING WHOLE NUMBERS

What Lies Ahead

In this section you will learn these skills.

- comparing whole numbers
- ordering whole numbers

In this section you will use these words.

- compare
- number line
- order
- greater than, less than, equal to
- increasing and decreasing order

Working Together

You compare and order whole numbers frequently.

When you **compare** whole numbers, you look for differences in place values.

To compare values, these symbols are used.

>	greater than
<	less than
=	is equal to

This section deals with comparing and ordering numbers.

Video Activity

To review place value, watch the video *MATH WORKS: Place Value of Large Numbers* (AIT).

If you cannot view the video, continue reading.

Comparing Numbers

Place value helps you compare numbers.

Example 1

Use the chart at the right to compare the food energy in the following.

- a serving of skim milk and a serving of plain cornflakes

$$\underset{\uparrow}{3}60 > \underset{\uparrow}{2}45$$

3 hundreds are greater than 2 hundreds.

- a potato and a serving of cheddar cheese

$$\underset{\uparrow}{3}80 < \underset{\uparrow}{7}60$$

3 hundreds are less than 7 hundreds.

- a bran muffin and a serving of skim milk.

$$360 = 360$$

Food Energy per Serving in Kilojoules (kJ)

Milk and Milk Products		Fruit and Vegetables	
Butter	450	Apple	290
Cheese, cheddar	760	Banana	420
Milk, skim	360	Broccoli	90
Milk, 2%	540	Cabbage	75
Milk, whole	660	Potato, boiled	380
Milkshake	1280	Strawberries	240
Meat and Meat Alternatives		**Breads and Cereals**	
Beef, roast	810	Bread, white	340
Cod	640	Bread, whole wheat	300
Chicken, fried	770	Cornflakes, plain	245
Egg, boiled	330	Muffin, bran	360
Hamburger patty	1080	Oatmeal	290
Pork, roast	890	Spaghetti	483

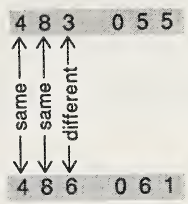
[1]

[1]Reproduced with permission of the Minister of Supply and Services Canada.

Example 2

Which is larger? 483 055 or 486 061?

To compare larger values, read the digits of both numbers **from the left** until you find a difference between two digits **in the same place**.



486 061 has 6 thousands, but 483 055 has only 3 thousands.

So, 486 061 > 483 055.
Read: "486 061 is greater than 483 055."
We can reverse the statement: 483 055 < 486 061.
Read: "483 055 is less than 486 061."

Note

- If one whole number has more digits than another, the number with more digits is greater. e.g. 50 024 > 8 935
- If all digits of two whole numbers are the same, the two numbers are **equal**. e.g. 1895 = 1895

Ordering Numbers

Here are three test marks: 76%, 82%, 73%.

Order the marks from highest to lowest (greatest to least).

Compare the tens first. 82 has 8 tens; the others have 7 tens. So, 82 is highest.

76
82
73
↑

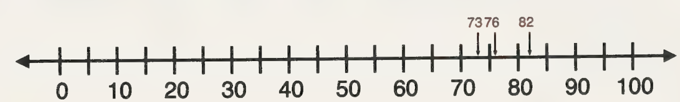
Compare the ones in 76 and 73. 76 has 6 ones; 73 has 3 ones. So, 76 is higher than 73.

76
73
↑

The order of marks from highest to lowest is 82%, 76%, 73%.

Note

A **number line** can help you to order numbers.

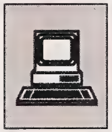


Numbers increase as you move from left to right.

Practice Activities

Space for Your Work

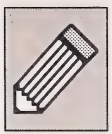
Computer Alternative



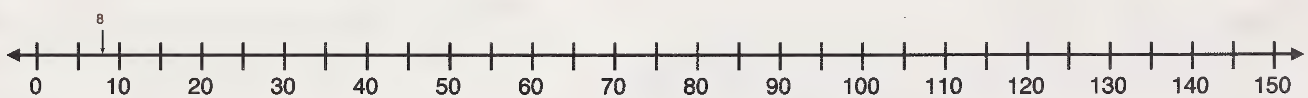
1. If you can use a computer, do Lesson 2 of the "Numbers and Numeration" disk from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions in the folder with the disk before using the program. Remember, if you need help or get a question wrong, hold down the SHIFT key and press the ? key.

Print Alternative



2. Place the numbers 8, 29, 66, 102, and 140 on the number line below. The 8 has been placed on the number line as an example.

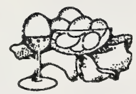


For Questions 3-4, use the food chart in this section.

Space for Your Work

3. Compare the energy content of single servings of these foods using < or >.

   a. skim milk and 2% milk 

   b. boiled egg and a boiled potato 

   c. white bread and whole wheat bread 

   d. a banana and an apple 

4. For breakfast, Hans had single servings of cornflakes, banana, 2% milk, and whole wheat toast with butter. List the energy content of these foods from greatest to least.

5. Arrange from least to greatest.

   a. 178, 59, 765, 384, 43

   b. 623, 528, 856, 365, 563

   c. 929, 299, 292, 229, 922

Space for Your Work

6. Order all the numbers from least to greatest.

55 082	59 652	35 007	51 908
54 658	52 662	56 062	57 954
58 901	50 657	59 182	54 366

7. The chart below shows the number of farms in each province in 1986. Order the provinces from the greatest number of farms to the least number of farms.

Number of Farms	
Newfoundland	651
Prince Edward Island	2 833
Nova Scotia	4 283
New Brunswick	3 554
Quebec	41 448
Ontario	72 713
Manitoba	27 336
Saskatchewan	63 431
Alberta	57 777
British Columbia	19 063

[1]

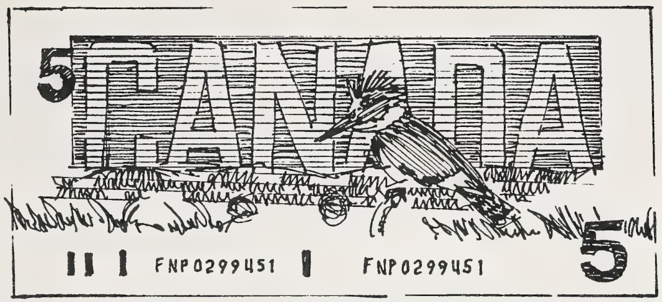
See your learning facilitator to check your answers and to receive further instructions.

[1]Statistics Canada

Concluding Activities

Space for Your Work

1. Compare the serial numbers on two or more five-dollar bills. Tell which was printed first.



2. Counting numbers are arranged in four columns as shown. Under which letter will the number 101 appear?

A	B	C	D
1	2	3	4
8	7	6	5
9	10	11	12
16	15	14	13

[1]

[1]Alberta Education for the excerpt from *Problem Solving Challenge for Mathematics,* Edmonton, 1985.

3. Orest has 3 dogs. The largest dog is 3 kg heavier than the medium-size dog. The medium-size dog is 4 kg heavier than the smallest dog. Together the three dogs have a mass of 29 kg. What is the mass of each dog?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

ROUNDING WHOLE NUMBERS

What Lies Ahead

In this section you will review these skills.

- rounding whole numbers
- identifying rounded and exact numbers

In this section you will use these words.

- round
- rounded number

Working Together

Some quantities, like populations, change so often that exact numbers are hard to determine.

Often, exact numbers may not be required. Approximate numbers may be used.

Rounding numbers makes them approximate. This section deals with rounding.

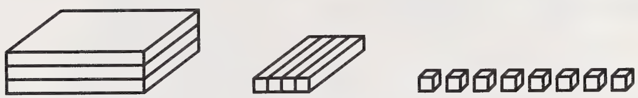
Using Base 10 Blocks to Round

Base 10 blocks can be helpful when first learning to rounding numbers because they help you see the relationship.

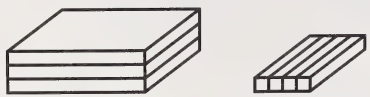
Example

Round 347 to the nearest tenth.

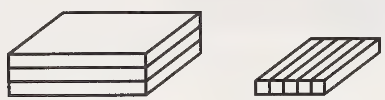
Is 347 closer to 340 or 350?



347



340



350

You must take away 7 units from 347 to make 340.

You must add 3 units to 347 to make 350.

So, 347 when rounded to the nearest 10 is 350.

Using Number Lines to Round

Number lines are also helpful when first learning to round numbers because you can see the relationship.

Example

Round 347 to the nearest 10.

Is 347 closer to 340 or 350?

You can see that 347 is closer to 350 than to 340.

So, 347 rounded to the nearest 10 is 350.

Rounding Rules

You can use rules when rounding numbers. See the following three examples.

Example 1

The 1986 official road map of the Province of Alberta gave the population of Edmonton as 560 085 and the population of Calgary as 625 143. Round these numbers to the nearest ten thousand.

Solution

Rules	Edmonton	Calgary
Step 1: Find the rounding place.	5 6 0 0 8 5 ↑ (under 6)	6 2 5 1 4 3 ↑ (under 2)
Step 2: Check the place to the right of the rounding place.	5 6 0 0 8 5	6 2 5 1 4 3
Step 3: If the digit to the right of the rounding place is less than 5, leave the digit in the rounding place as is, and change all the digits to its right to 0.	5 6 0 0 0 0	
Step 4: If the digit to the right of the rounding place is 5 or more, increase the digit in the rounding place by 1, and change all the digits to its right to 0.		6 3 0 0 0 0

Rounded to the nearest ten thousand, the population of Edmonton is 560 000, and the population of Calgary is 630 000.

Example 2

Round 5472 to the nearest hundred.

Step 1: Find the hundreds digit.

$$\underset{\substack{\uparrow\\\text{hundreds digit}}}{5\ 4}\ 7\ 2$$

Step 2: Look at the digit to the right of the hundreds digit.

$$\underset{\substack{\uparrow\\\text{hundreds digit}}}{5\ 4}\ 7\ 2$$

Step 3: Is this digit 5 or more? Yes.

Increase the hundreds digit by 1, and change all the digits to the right of the hundreds digit to 0.

$$\underset{\uparrow}{5\ 5}\ 0\ 0$$

Note

When a 9 is increased, regrouping is necessary.

e.g. 8296 rounded to the nearest ten becomes 8300 after regrouping the tens to the hundreds.

e.g. 985 rounded to the nearest hundred becomes 1000.

Example 3

Round 664 728 to the nearest ten thousand.

Step 1: Find the ten thousands digit.

$$\underset{\substack{\uparrow\\\text{ten thousands digit}}}{6\ 6}\ 4\ \ 7\ 2\ 8$$

Step 2: Look at the digit to the right of the ten thousands digit.

$$\underset{\substack{\uparrow\\\text{ten thousands digit}}}{6\ 6}\ 4\ \ 7\ 2\ 8$$

Step 3: Is this digit 5 or more? No. The digit is less than 5.

Leave the ten thousands digit unchanged and change all the digits to the right of the ten thousands digit to 0.

$$\underset{\uparrow}{6\ 6}\ 0\ \ 0\ 0\ 0$$

Practice Activities

Space for Your Work

1. Answer true (T) or false (F).

   a. The number for A is closer to 500 than to 600.

   b. The number for B is closer to 500 than 600.

   c. The number for C is closer to 700 than 800.

   d. The number for D is closer to 700 than 800.

   e. The number for E is closer to 700 than 800.

2. The number in the middle is closer to one of the outside numbers than the other. Tell which number it is closer to. One has been done for you as an example.

   a. 30, 32, 40

   b. 80, 85, 90

   c. 200, 219, 300

   d. 600, 651, 700

   e. 30 000, 37 215, 40 000

   f. 900 000, 968 200, 1 000 000

2. a. 30

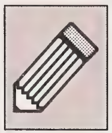
Computer Alternative

Space for Your Work

3. If you can use a computer, do Lessons 6, 7, and 8 of the disk "Numbers and Numeration" from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions in the folder with the disk before you use the program. If you need help or get a question wrong, hold down the SHIFT key and press the ? key.

Print Alternative



4. Round to the nearest thousand.
   a. 6071
   b. 5982

5. Round to the nearest hundred thousand.
   a. 784 300
   b. 452 121

6. Round to the nearest ten million.
   a. 9 825 850
   b. 120 980 000

Space for Your Work

7. The following chart shows the number of tourist visits to Canada. Round each number in the chart to three places.

   a. the nearest ten thousand

   b. the nearest hundred thousand

Tourist Visits to Canada		rounded to nearest ten thousand	rounded to nearest hundred thousand
United States	12 499 000		
Europe	892 322		
Asia	196 439		
Caribbean	110 721		
Australasia	55 508		
South America	46 370		
Africa	26 196		

[1]

See your learning facilitator to check your answers and to receive further instructions.

[1]Statistics Canada

Concluding Activities

Space for Your Work

Jack and his father are shopping. His father has a limited amount of money which he cannot overspend. He has no calculator, so he is rounding prices and adding as he goes along. Jack's father rounds every price to the highest dollar, even if the price is closer to the lowest dollar (e.g. \$2.29 ≐ \$3.00, instead of \$2.00).

1. Why do you think Jack's father always rounds up? What is the advantage of this method?

2. What is the disadvantage of this method?

> See your learning facilitator to check your answers and to receive further instructions.

[1]Statistics Canada

ESTIMATING SUMS

What Lies Ahead

In this section you will learn these skills.

- deciding when to make estimates
- estimating whole number sums

In this section you will use these words.

- estimate
- estimation

Working Together

This section deals with an important skill — estimating.

Many everyday situations require the use of estimating skills. You must decide whether it is necessary to make an estimate, to calculate the answer, or to do both.

Video Activity

View the video *THINK ABOUT: Using Estimating and Approximating* (AIT). Look for situations in which the characters estimate or calculate how they make these choices. Then read the examples in this section.

If you cannot view the video, read the program summary. Then read the examples in this section.

Program Summary

Vickie, David, and their grandmother decide to insulate the farmhouse.

They use estimation before purchasing insulation and installing plastic storm windows.

Strategies for Estimating Sums

There are three strategies to help you estimate sums. These strategies are rounding, betweenness, and front-end digits.

Remember that unlike the strategies for mental computation, these estimating strategies will **not** give an exact answer.

Remember that estimating is done mentally.

Rounding

A common strategy for estimating sums is to round the addends and find the rounded sum.

Example: Estimate 382 + 160 + 299

$$\begin{aligned} 382 + 160 + 299 &\doteq 400 + 200 + 300 \\ &\doteq 900 \end{aligned}$$

With several numbers or larger numbers, it can be helpful to round all addends to the same place.

In this example all the addends were rounded to the nearest 100.

Rounding to a smaller place usually gives a more accurate estimate, but makes the adding more difficult. You must decide how accurate an estimate you need.

Note

The symbol $\doteq$ means "is about equal to."

Usually when rounding, you should use the rules that you learned in Section 4 of this module. However, sometimes it is desirable to find the smallest estimate and the highest estimate.

- To find the **smallest** estimate you should round all the addends down.

Example: Estimate 54 + 82 + 37

$$\begin{aligned} 54 + 82 + 37 &\doteq 50 + 80 + 30 \\ &\doteq 160 \end{aligned}$$

- To find the **highest** estimate you should round all the addends up.

Example

$$\begin{aligned} 54 + 82 + 37 &\doteq 60 + 90 + 40 \\ &\doteq 190 \end{aligned}$$

The exact answer will be between these two estimates.

Using Front-End Digits

Another method used in estimating sums is using front-end digits. In this method you only use the front digits. You replace the other digits with zeros.

Example: Estimate 856 + 39 + 476

$$\begin{aligned} 856 + 39 + 476 &\doteq 800 + 30 + 400 \\ &\doteq 1230 \end{aligned}$$

Note

This strategy always produces low estimates because you are actually rounding down.

Practice Activities

Space for Your Work

Do either Question 1 or 2.

1. If you watched the video, answer the following questions about the video, "Using Estimating. and Approximating."

   a. Describe two situations in the story in which approximate numbers were used.

   b. Name two things that Vickie and David measured by estimating.

   c. At the hardware store, why did Grandma, Vickie, and David decide to go back home and measure the attic?

If you did not watch the video, do the following.

2. A newspaper headline reads "25 000 Attend Playoff Game". Is an estimate sufficient in these situations?

   a. The accountant figures out how much money was made on ticket sales.

   b. The newspaper reports the number of people who attended the game.

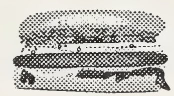
Space for Your Work

3. Tell whether the number is exact or approximate.

   a. A hockey player scored 92 goals in one season.

   b. The apartment rent is $356 per month.

   c. Fifty-five million hamburgers have been sold.



   d. The town has a population of 4000.

   e. The Wolds live 172 km from Edmonton.

4. Tell the number of digits in the sums.

   a. 38 + 46

   b. 825 + 36

   c. 1284 + 32

   d. 356 + 248

Space for Your Work

5. Estimate the sums.

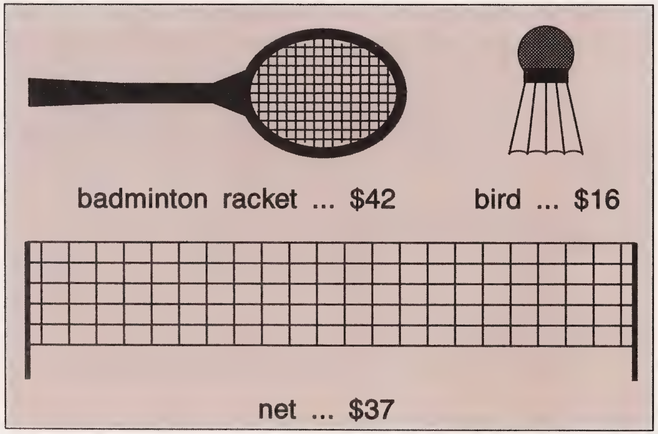
a. $62 + 27$

b.
$$\begin{array}{r} 635 \\ +\ 7049 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 921 \\ 367 \\ 405 \\ +\ 883 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 35\,679\,002 \\ +\ 29\,941\,750 \\ \hline \end{array}$$

6. Estimate the cost of the badminton equipment.



7. Use the chart below. Estimate the total number of farms.

Number of Farms	
Newfoundland	651
Prince Edward Island	2 833
Nova Scotia	4 283
New Brunswick	3 554
Quebec	41 448
Ontario	72 713
Manitoba	27 336
Saskatchewan	63 431
Alberta	57 777
British Columbia	19 063

[1]

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

[1]Statistics Canada

Extra Practice

Space for Your Work

1. Estimate the sums. Fill in the letter of the line to which each sum belongs. One has been done as an example.

   a. 200 + 840

   b. 654 + 87

   c. 982 + 1025

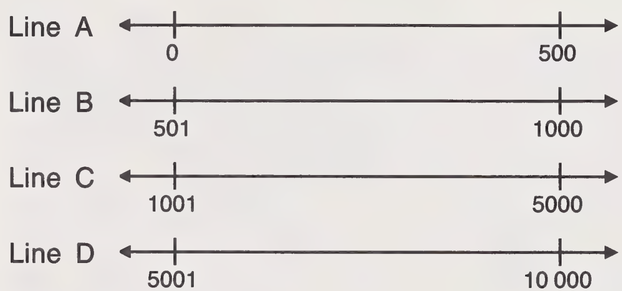
   d. 146 + 283

   e. 845 + 4525

   f. 96 + 1025

   g. 999 + 3

   h. 2042 + 4650



[1]

1. a. Line C.

[1]National Council of Teachers of Mathematics for excerpts from *Ideas from the Arithmetic Teacher,* Reston, Virginia, 1979.

Space for Your Work

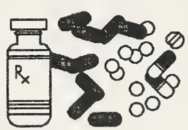
2. In each case, tell if an exact number would be needed, or if an estimate would be good enough.

   a. You want to calculate pay cheques.

   b. You want to figure out the number of hot dogs needed for a picnic.

   c. You want to find the distance to be travelled on a vacation trip.

   d. You want to find a medicine dosage.



See your learning facilitator to check your answers and to receive further instructions.

Working Together

Compensating

Compensating (adjusting) is another method used in estimating sums.

You can use compensating with rounding or with front-end digits.

Example 1: Estimate 382 + 160 + 299 using rounding and compensating.

$$\begin{aligned} 382 + 160 + 299 &\doteq 400 + 200 + 300 \\ &\doteq 900 \end{aligned}$$

All the numbers were rounded up so this estimate is too high. Compensate (adjust) your estimate.

$$382 + 160 + 299 \doteq 870$$

Example 2: Estimate 856 + 39 + 476 using front-end digits and compensating.

$$\begin{aligned} 856 + 39 + 476 &\doteq 800 + 30 + 400 \\ &\doteq 1230 \end{aligned}$$

This estimate is low. Compensate (adjust your estimate).

$$856 + 39 + 476 \doteq 1350$$

Concluding Activities

Space for Your Work

1. Circle the most accurate estimate from the choices given at the right.

   a. 83 + 49

   b. 152 + 108

   c. 32 460 + 5 666

   d. $\begin{array}{r} 819 \\ 273 \\ +\ 590 \\ \hline \end{array}$

1. a. 80, 100, 130, 150

   b. 150, 200, 250, 300

   c. 32 000, 35 000, 38 000, 40 000

   d. 1600, 1700, 1800, 2000

2. Estimate the sums of the following. Predict whether the exact sum will be more or less than the estimate.

   a. 711 + 623

   b. $\begin{array}{r} 6470 \\ +\ 2830 \\ \hline \end{array}$

   c. $\begin{array}{r} 78\,420 \\ +\ 56\,311 \\ \hline \end{array}$

See your learning facilitator to check your answers and to receive further instructions.

ESTIMATING DIFFERENCES

What Lies Ahead

In this section you will learn these skills.

- estimating whole number differences
- determining whether a calculated difference is reasonable or whether an error was made

In this section you will use these words.

- estimate
- round
- minuend
- subtrahend
- difference

Working Together

In this section you will use estimation skills to produce approximate differences. You will also check the reasonableness of exact subtraction calculations or whether an error in calculation was made.

This section will discuss two strategies for estimating differences — rounding and using front-end digits.

Rounding

A common strategy for estimating differences is rounding.

Example Estimate 2873 − 1639

You can round to the nearest 1000.

$$\begin{aligned} 2873 - 1639 &\doteq 3000 - 2000 \\ &\doteq 1000 \end{aligned}$$

— OR —

You can round to the nearest 100.

$$\begin{aligned} 2873 - 1639 &\doteq 2900 - 1600 \\ &\doteq 1300 \end{aligned}$$

Using Front-End Digits

Another common strategy for estimating differences is using front-end digits.

Example: Estimate 2873 − 1639

$$\begin{aligned} 2873 - 1639 &\doteq 2000 - 1000 \\ &\doteq 1000 \end{aligned}$$

When subtracting numbers that have the same front digits, the first 2 digits from each number may have to be used.

Example: Estimate 39 742 − 34 815

$$\begin{aligned} 39\,742 - 34\,815 &\doteq 39\,000 - 34\,000 \\ &\doteq 5000 \end{aligned}$$

Practice Activities

Space for Your Work

1. Circle the most accurate estimate from the choices given at the right.

   a. 85 – 48

   b. 693 – 485

   c. 53 240 – 14 881

   d. $\begin{array}{r} 65\,523\,700 \\ -\ \ 8\,250\,194 \\ \hline \end{array}$

1. a. 50, 60, 70, 100

   b. 100, 150, 200, 300

   c. 38 000, 39 000, 40 000, 45 000

   d. 55 000 000, 57 000 000, 58 000 000, 6 000 000

2. Tell whether the difference will be more or less than 500.

   a. $\begin{array}{r} 630 \\ -\ 185 \\ \hline \end{array}$

   b. $\begin{array}{r} 741 \\ -\ 369 \\ \hline \end{array}$

   c. $\begin{array}{r} 309\,477 \\ -\ 308\,995 \\ \hline \end{array}$

   d. 5789 – 5190

Space for Your Work

3. Estimate the difference and predict whether the exact difference will be more or less than the estimate.

   a. $\begin{array}{r} 326 \\ -\ 189 \\ \hline \end{array}$

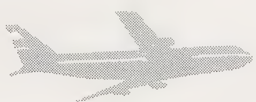
   b. $\begin{array}{r} 741 \\ -\ 369 \\ \hline \end{array}$

   c. $\begin{array}{r} 309\,477 \\ -\ 308\,995 \\ \hline \end{array}$

   d. 607 – 339

   e. 41 777 – 28 666

4. If 376 out of 2145 employees of an airline are pilots, estimate the number of employees who are not pilots.



See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Estimate the differences.

   a. $\begin{array}{r} 91 \\ -\ 47 \\ \hline \end{array}$

   b. $\begin{array}{r} 246 \\ -\ 85 \\ \hline \end{array}$

   c. $\begin{array}{r} 5\,174\,808 \\ -\ 2\,606\,283 \\ \hline \end{array}$

   d. 510 – 136

   e. 4491 – 912

2. In Canada 15 334 000 people have English for their mother tongue, while 6 160 000 have French. Estimate how many more people have English for a mother tongue.

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Just by estimating, tell which answers cannot be correct.

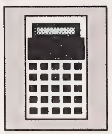
   a.
   $$\begin{array}{r} 885 \\ -\ 497 \\ \hline 588 \end{array}$$

   b. 9450 – 4888 = 4562

   c. 15 784 – 10 631 = 5153

   d.
   $$\begin{array}{r} 169\,000\,500 \\ -\ \ 24\,500\,000 \\ \hline 134\,500\,500 \end{array}$$

2. Check the answers to Question 1 with a calculator.



See your learning facilitator to check your answers and to receive further instructions.

ESTIMATING PRODUCTS

What Lies Ahead

In this section you will learn these skills.

- estimating whole number products
- determining if a calculated product is reasonable, or whether an error was made

In this section you will use these words.

- estimate
- round
- multiplier
- multiplicand
- product
- factors
- range

Working Together

In this section you will compute approximate products, you will also determine if calculated products are reasonable, or whether an error was made. You will examine three strategies for estimating products — rounding, betweenness, and using front-end digits.

Video Activity

View the video *SOLVE IT: Estimation Strategies for Multiplication* (AIT) to find out how three boys use estimation skills to help themselves on a canoe trip. Then read the strategies for estimating products.

If you cannot view the video, study the strategies for estimating products in this section.

Rounding

You can round one factor or both factors and then find the rounded product.

Example: Estimate 8×57

You can round 57 to 60.

$$\begin{aligned} 8 \times 57 &\doteq 8 \times 60 \\ &\doteq 480 \end{aligned}$$

Example: Estimate 68×37

You can round both 68 and 37 to the nearest 10.

$$\begin{aligned} 68 \times 37 &\doteq 70 \times 40 \\ &\doteq 2800 \end{aligned}$$

Note

- You can round one or both factors.
- Estimating is done mentally so use numbers which are easy to work with.

Betweenness

It is often helpful to find a range of estimates.

You can round one of the factors up, and find the rounded product.

Then you can round the factor down, and find the rounded product. The exact answer will be between these two estimates.

Example: Estimate 4×86

Round 86 down.

$$\begin{aligned} 4 \times 86 &\doteq 4 \times 80 \\ &\doteq 320 \end{aligned}$$

Round 86 up.

$$\begin{aligned} 4 \times 86 &\doteq 4 \times 90 \\ &\doteq 360 \end{aligned}$$

4×86 is between 320 and 360.

Example 2: Estimate 93×9

Round 93 down.

$$\begin{aligned} 93 \times 9 &\doteq 90 \times 9 \\ &\doteq 810 \end{aligned}$$

Round 93 up.

$$\begin{aligned} 93 \times 9 &\doteq 100 \times 9 \\ &\doteq 900 \end{aligned}$$

93×9 is between 810 and 900.

Using Front-End Digits

You can multiply only the front-end digits of the factors and add the required number of zeros. The number of zeros to be added equals the number of remaining digits in the factors.

Example 1: Estimate 72×47

$$\underbrace{72 \times 47}_{\text{front digits}} \doteq 70 \times 40$$
$$\doteq 7 \times 4 + 2 \text{ zeros}$$
$$\doteq 28 + 2 \text{ zeros}$$
$$\doteq 2800$$

Example 2: Estimate 48×531

$$\underbrace{48 \times 531}_{\text{front digits}} \doteq 40 \times 500$$
$$\doteq 4 \times 5 + 3 \text{ zeros}$$
$$\doteq 20 + 3 \text{ zeros}$$
$$\doteq 20\,000$$

Note

- This strategy can be used with large numbers.
- This strategy always produces **low** estimates.

Compensating

You can use compensating with front-end digits and when both factors are rounded in the same direction.

Example: Estimate 48×531 using front-end digits and compensating.

$$48 \times 531 \doteq 40 \times 500$$
$$\doteq 20\,000$$

This estimate is low so adjust your estimate.

$$48 \times 531 \doteq 25\,000$$

Example: Estimate 68×37 using rounding and compensating.

$$68 \times 37 \doteq 70 \times 40$$
$$\doteq 2800$$

This estimate is high so adjust your estimate.

$$68 \times 37 \doteq 2500$$

Practice Activities

Space for Your Work

Do either Question 1 or 2.

1. If you watched the video program do the following.

   a. Name two estimations made by the boys.

   b. Describe the three estimation strategies by the boys.

   c. Why did the boys use estimations rather than exact calculations?

2. If you did not watch the video, give examples in everyday life when an exact product is needed and when an estimated product is sufficient.

3. How many digits will be in the product?

   a. 230×3

   b. 18×26

   c. 1850×49

   d. 786×952

4. Circle the most accurate estimate listed at the right. Then predict whether the actual product will be more or less.

   a. 62×29

   b. 19×73

   c. 52×49

5. Round the second factor to the nearest ten, hundred, or thousand, to make it easy to multiply. Then multiply mentally to get an estimate. One is done as an example.

   a. 5×874 is approximately 4500 .

   b. 3×57 is approximately ______ .

   c. 6×42 is approximately ______ .

   d. 8×233 is approximately ______ .

   e. 6×789 is approximately ______ .

   f. 3×2350 is approximately ______ .

Space for Your Work

4. a. 1200, 1400, 1800, 2100

   b. 700, 800, 1400, 1600

   c. 2000, 2400, 2500, 3000

Space for Your Work

6. When using betweenness to estimate in multiplication, round both up and down to find estimate boundaries. One is done as an example.

   a. 8 × 542 is between __4000__ and __4800__ .

   b. 4 × 429 is between ________ and ________ .

   c. 7 × 3840 is between ________ and ________ .

   d. 3 × 256 is between ________ and ________ .

   e. 2 × 850 is between ________ and ________ .

7. Estimate by multiplying the front-end digits and by using place value to add the correct number of zeros. One is done as an example.

   a. 34 × 219 is approximately __6000__ .

   b. 310 × 220 is approximately ________ .

   c. 72 × 53 214 is approximately ________ .

   d. 68 × 341 is approximately ________ .

   e. 524 × 415 is approximately ________ .

Space for Your Work

8. According to a map of the river, marker buoys are 450 m apart. If the teenagers need to paddle their canoe past 7 more buoys before they arrive at the waterfall, about how far is it to the waterfall? Give upper and lower estimates.

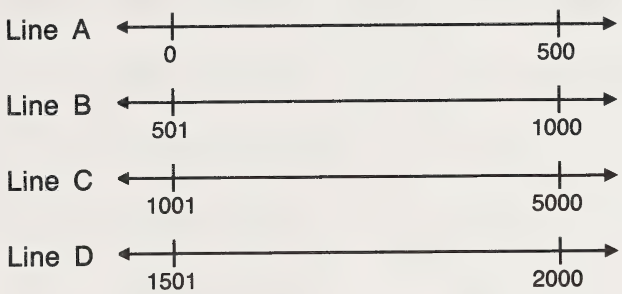
9. On the map, the distance from Seneca Falls to the fork of White River is 228 mm. Each mm on the map represents 80 m of actual distance. About how many metres is it from Seneca Falls to White River?

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Estimate the products. Fill in the letter of the line to which each product belongs. One has been done as an example.



1. 9×8	5. 45×33	9. 14×8
2. 25×12	6. 18×72	10. 32×61
3. 51×12	7. 35×16	11. 87×8
4. 21×40	8. 105×12	12. 54×23

[1]

1. A

See your learning facilitator to check your answers and to receive further instructions.

[1]National Council of Teachers of Mathematics for excerpts from *Ideas from the Arithmetic Teacher*, Reston, Virginia, 1979.

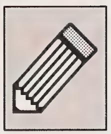
Concluding Activities

Space for Your Work

Computer Alternative

1. Do the program, "Multi Targets" on Disk A of MAC 6. Information on the program is in the folder with the disk.

Print Alternative



2. a. Place the numbers 1, 2, 3, 4, and 5 in the boxes to make the largest possible product and the smallest possible product. (Use your calculator to help you decide.)

   b. Now try these numbers: 5, 2, 4, 6, 0.

   c. Now try these numbers: 8, 9, 0, 4, 3.

	Largest Possible Product	Smallest Possible Product
2. a.	$\begin{array}{r} \square\square\square \\ \times\ \square\square \\ \hline \end{array}$	$\begin{array}{r} \square\square\square \\ \times\ \square\square \\ \hline \end{array}$
b.	$\begin{array}{r} \square\square\square \\ \times\ \square\square \\ \hline \end{array}$	$\begin{array}{r} \square\square\square \\ \times\ \square\square \\ \hline \end{array}$
c.	$\begin{array}{r} \square\square\square \\ \times\ \square\square \\ \hline \end{array}$	$\begin{array}{r} \square\square\square \\ \times\ \square\square \\ \hline \end{array}$

d. Now try these numbers: 6, 2, 4, 3, 8.

e. Now try these numbers: 2, 7, 6, 1, 8.

f. Did you discover a pattern for the largest product? the smallest product? What is the pattern?[1]

Space for Your Work

	Largest Possible Product	Smallest Possible Product
d.	$\square\square\square \times \square\square$	$\square\square\square \times \square\square$
e.	$\square\square\square \times \square\square$	$\square\square\square \times \square\square$

See your learning facilitator to check your answers and to receive further instructions.

[1]National Council of Teachers of Mathematics for excerpts from *The Arithmetic Teacher*, Reston, Virginia, October, 1987.

ESTIMATING QUOTIENTS

What Lies Ahead

In this section you will learn these skills.

- estimating whole number quotients
- determining if calculated whole number quotients are reasonable, or whether an error was made

In this section you will use these words.

- estimate
- round
- divisor
- dividend
- quotient
- range

Working Together

Estimation strategies for division are similar to those for multiplication. This section deals with estimating quotients. There are two strategies for estimating quotients — using friendly numbers and using front-end digits.

Video Activity

Please view the video, *SOLVE IT: Estimation Strategies for Division* (AIT) to discover how two students use estimation to help them raise funds for their team.

If you cannot view the video, read the strategies for estimating quotients in this section.

Using Friendly Numbers

One way to estimate quotients is to round the dividend or divisor, or both, to numbers that you can divide easily.

Example 1: Estimate $714 \div 8$

Round 714 to 720.

$$\begin{aligned} 714 \div 8 &\doteq 720 \div 8 \\ &\doteq 90 \end{aligned}$$

Example 2: Estimate $16\overline{)2371}$

Round 16 to 20. Round 2371 to 2400.

$$\begin{aligned} 16\overline{)2371} &\doteq 20\overline{)2400} \\ &\doteq 120 \end{aligned}$$

Note

- Round so that the dividend can be divided **evenly** by the divisor.
- Estimation is done mentally.
- Using friendly numbers is sometimes called using **compatible numbers**.

Using Front-End Digits

Another way to estimate quotients is to divide only by the first one or two digits of the dividend by the first digit of the divisor. After that, add the required number of zeros to the quotient. Use special quotients to find the required number of zeros.

Example 1: Estimate $825 \div 4$

$$\begin{aligned} 825 \div 4 &\doteq 800 \div 4 \\ &\doteq 8 \div 4 + 2 \text{ zeros} \\ &\doteq 2 + 2 \text{ zeros} \\ &\doteq 200 \end{aligned}$$

Example 2: Estimate $53\overline{)22\,167}$

$$\begin{aligned} 53\overline{)22\,167} &\doteq 50\overline{)22\,000} \\ &\doteq 5\overline{)2200} \leftarrow 5\cancel{0}\overline{)22\,00\cancel{0}} \\ &\doteq 5\overline{)22} + 2 \text{ zeros} \\ &\doteq 4 + 2 \text{ zeros} \\ &\doteq 400 \end{aligned}$$

Note

- The front-end digits do **not** have to divide evenly.

Practice Activities

Space for Your Work

Do Question 1 or 2.

1. a. In the video, you were introduced to a manager of a sports stadium. Name two ways in which the manager uses estimation in his daily work.

   b. Describe how Pete estimates the number of chocolate bars he and Benny must sell per hour.

2. If you did not watch the video, describe situations in everyday life in which you might estimate quotients.

3. How many digits will be in the quotients?

   a. $832 \div 41$

   b. $6178 \div 52$

   c. $1500 \div 21$

   d. $38\,725 \div 95$

   e. $43\,872 \div 38$

4. Circle the best estimate from the choices given at the right.

a. $268 \div 3$

b. $500 \div 21$

c. $763 \div 11$

d. $387 \div 97$

5. Circle the front digits of the divisor and dividend you will use to estimate. Then write the estimate. One is done as an example.

a. 4652 $\div$ 53

b. 71 345 $\div$ 21

c. 6189 $\div$ 32

d. 65 102 $\div$ 81

e. 2685 $\div$ 32

Space for Your Work

4. a. 8, 80, 800, 8000

b. 2, 20, 200, 2000

c. 7, 70, 700, 7000

d. 4, 40, 400, 4000

5. a. The estimate is 90.

6. Write a related division problem using friendly numbers. Use it to give an estimate. One is done as an example.

   a. $268 \div 3$

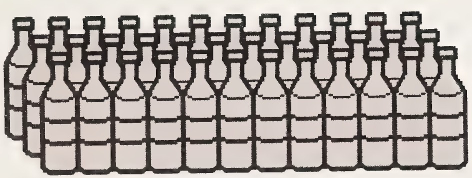
   b. $200 \div 7$

   c. $7150 \div 9$

   d. $137 \div 12$

   e. $152 \div 40$

7. In a month 26 teenagers collected 1386 bottles for recycling. Estimate the number of bottles collected by each teenager.



See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

6. a. $270 \div 3 = 90$

Extra Practice

Space for Your Work

1. Predict the number of digits in the quotient.

   a. 498 ÷ 30
   b. 1116 ÷ 80
   c. 41 623 ÷ 50
   d. 53 821 ÷ 25
   e. 498 ÷ 57

2. Circle the most accurate estimate from the choices given at the right.

   a. $53\overline{)2009}$
   b. $24\overline{)10\,247}$
   c. $42\overline{)12\,875}$

2 a. 4, 40, 400, 4000
b. 40, 50, 400, 500
c. 3, 30, 300, 3000

3. Are the quotients reasonable?

   a. 512 ÷ 8 = 64
   b. 7264 ÷ 33 = 121
   c. 68 085 ÷ 765 = 89
   d. 35 636 ÷ 59 = 6040

See your learning facilitator to check your answers and to receive further instructions.

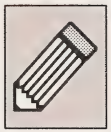
Concluding Activities

Space for Your Work

Computer Alternative

1. If you have a computer, do the program "Tug of War" on Disk A of MAC 6. Information on the program is in the folder with the disk.

Print Alternative



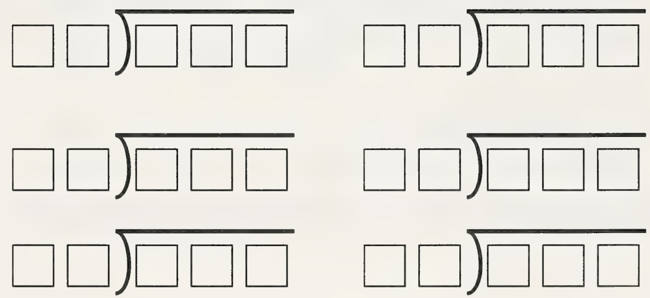
2. a. Put the numbers 1, 2, 3, 4, and 5 into the boxes at the right and make the largest possible quotient and the smallest possible quotient. (Use your calculator to help you decide.)

   b. Now try these numbers: 2, 4, 6, 8, 0.

   c. Now try these numbers: 8, 9, 0, 4, 3.

   d. Did you discover a pattern for the largest quotient and the smallest quotient? What is the pattern for each quotient?

Largest Possible Quotient　　Smallest Possible Quotient



See your learning facilitator to check your answers and to receive further instructions.

FINDING SUMS

What Lies Ahead

In this section you will review these skills.

- computing exact whole number sums
- checking the accuracy of whole number sums

In this section you will use these words.

- sum
- regrouping
- carrying

Working Together

In this section you will find exact sums on paper and with the use of a calculator.

Learning Aids Activities

To prepare yourself for this activity you will first investigate finding sums with concrete models. You will need base 10 blocks for the Learning Aids Activities.

Turn to Exercise B in the *Learning Aids Booklet* and do the activities for this section.

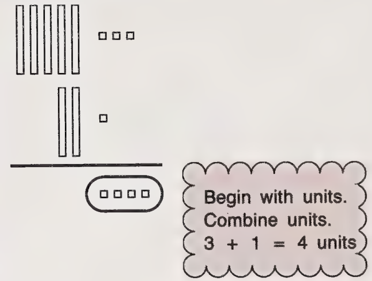
Working Together

Going from Concrete Models to Paper-and-Pencil Methods

Compare the steps in the paper-and-pencil methods with the concrete models.

Example 1: 53 + 21

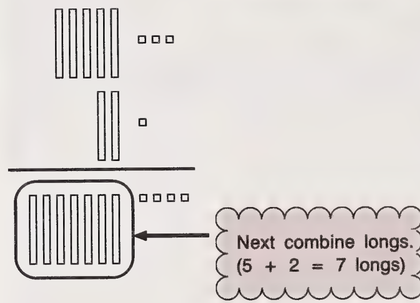
Concrete Model



Paper and Pencil

$$\begin{array}{r} 53 \\ +\ 21 \\ \hline 4 \end{array}$$

Add ones.



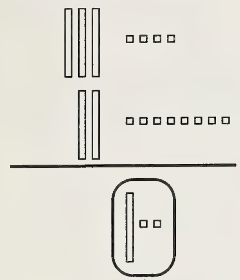
$$\begin{array}{r} 53 \\ +\ 21 \\ \hline 74 \end{array}$$

Add tens.

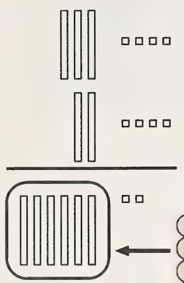
So, 53 + 21 = 74

Example 2: $34 + 28$

Concrete Model



Begin with units. Combine units.
(4 + 8 = 12 units)
Regroup 10 units for 1 long.
(This leaves 2 units.)



Combine longs.
(Altogether there
are 6 longs.)

Paper and Pencil

$$\begin{array}{r} {}^{1} \\ 34 \\ +\ 28 \\ \hline 2 \end{array}$$

Add ones.
Carry 1 ten.

$$\begin{array}{r} {}^{1} \\ 34 \\ +\ 28 \\ \hline 62 \end{array}$$

Add tens.

So, $34 + 28 = 62$.

Regrouping

Sometimes regrouping is necessary when you add. If a column totals 10 or more, move a 1 for every 10 to the next higher column.

Regrouping is sometimes called **carrying**. It is helpful to write the carrying numbers above the columns to which they are moved, but you can regroup mentally if you wish.

Example 1: Add 4289 and 3061.

Solution

$$\begin{array}{rl} 1\,1 & \leftarrow \text{regrouping or carrying numbers} \\ 4289 & \\ +\ 3061 & \\ \hline 7340 & \end{array}$$

Example 2: Find the sum of 7502 and 647.

Solution

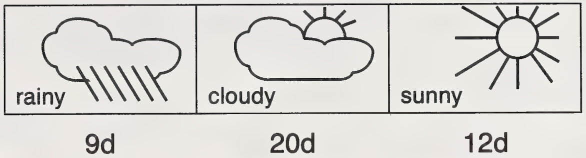
$$\begin{array}{rl} 1 & \leftarrow \text{regrouping or carrying numbers} \\ 7802 & \text{addend} \\ +\ \ 647 & \text{addend} \\ \hline 8449 & \text{sum} \end{array}$$

Practice Activities

Space for Your Work

Do Questions 1-3 without a calculator.

1. Rajeet observed weather conditions. How many days did she observe?



2. Find the sums.

   a. 2748 + 345 + 506

   b. 928 + 542 + 140

   c. 8075 + 430

   d. 999 + 22887

   e. 96 319 + 4284

   f. 268 451 + 917 449

Space for Your Work

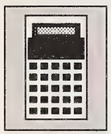
3. Fill in the missing digits.

a.

$$\begin{array}{r} \boxed{}\ 4\ 6\ 9 \\ +\ 7\ \boxed{}\ 2\ \boxed{} \\ \hline 1\ 0\ 9\ \boxed{}\ 3 \end{array}$$

b.

$$\begin{array}{r} \boxed{}\ 6\ 5 \\ +\ 6\ \boxed{}\ 8 \\ \hline 9\ 1\ \boxed{} \end{array}$$



4. Solve by using a calculator. The chart shows the population of four Alberta cities in 1986.

Population	
Calgary	636 104
Edmonton	573 982
Lethbridge	58 841
Red Deer	54 425

[1]

Find the total population of the four cities listed in the table.

[1] Statistics Canada

Computer Alternative

Space for Your Work

5. If you wish to have more instruction and practice in addition, try the computer program *Conquering Whole Numbers* (MECC). Addition is the first lesson. A description of the program and instructions for use are given in the folder with the disk.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Palindromes read the same forward or backward. Radar, deed, and pop are palindromes.

This saying of Napoleon's is a palindrome.

Able was I ere I saw Elba.

Numbers are also palindromes.

These numbers are palindromes.

1331   72 327

Consider this example.

Napoleon was born in 1769. By reversing and adding you can make a palindrome from his birth year.

Step 1: Reverse 1769 and add.

$$\begin{array}{r} 1769 \\ +\ \ 9671 \\ \hline 11\,440 \end{array}$$

11 440 is not a palindrome.

Step 2: Reverse 11 440 and add.

$$\begin{array}{r} 11\,440 \\ +\ 04\,411 \\ \hline 15\,851 \end{array}$$

15 851 is a palindrome.

It took 2 reversals to make Napoleon's birth year a palindrome.

Concluding Activities

Space for Your Work

Do **not** use a calculator on the following.

1. a. In what year were you born? ________
   How many reversals are needed to make a palindrome from your birth year?

   b. Try this with the birth years of other family members and some friends. Who has the most reversals?

2. Below are three sets of digits: 3 fives, 3 ones, and 3 nines. These make a total of nine digits. The object is to cross out six of the digits and leave three so that when added together you have a sum of 20. How can this be done?

   5 5 5 1 1 1 9 9 9[1]

See your learning facilitator to check your answers and to receive further instructions.

[1]Alberta Education for excerpt from *Problem Solving Challenge for Mathematics*, Edmonton, 1985.

FINDING DIFFERENCES

What Lies Ahead

In this section you will learn these skills.

- computing exact whole number differences
- checking the accuracy of calculated whole number differences

In this section you will use these words.

- minuend
- subtrahend
- difference
- regrouping
- borrowing

Working Together

In this section you will now find exact differences by using pencil and paper and by using a calculator.

Learning Aids Activities

You will begin by investigating subtraction using concrete models. You will need base 10 blocks for the Learning Aids Activities.

To prepare yourself for this section, turn to Exercise C in the *Learning Aids Booklet* and complete the activities.

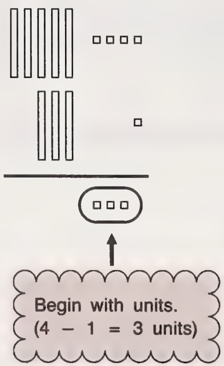
Working Together

Going from Concrete Models to Paper-and-Pencil Methods

Compare the steps in the paper and pencil methods with concrete models.

Example 1: 54 − 31

Concrete Model

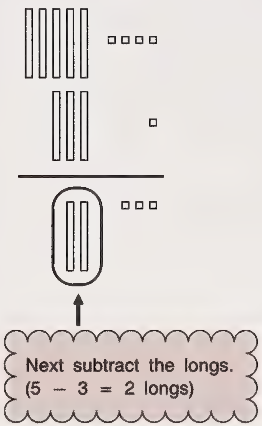


Paper and Pencil

$$\begin{array}{r} 54 \\ -\ 31 \\ \hline 3 \end{array}$$

Subtract the ones.

Concrete Model



Paper and Pencil

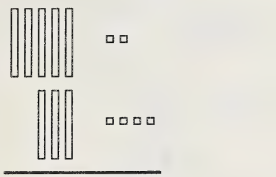
$$\begin{array}{r} 54 \\ -\ 31 \\ \hline 23 \end{array}$$

Subtract the tens.

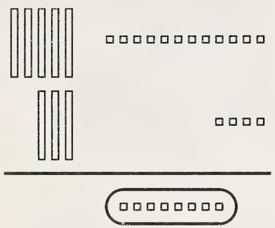
So, 54 − 31 = 23.

Example 2: 52 − 34

Concrete Model



Begin with units. You can't take 4 away from 2.



Trade 1 long for 10 units. Then take away 4 units from 12 units. (12 − 4 = 8 units)

Paper and Pencil

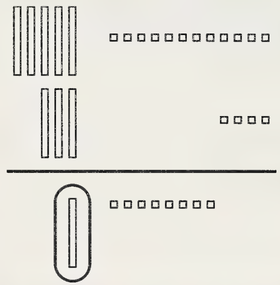
$$\begin{array}{r} 52 \\ -\ 34 \\ \hline \end{array}$$

Begin with ones. You can't subtract 4 from 2.

$$\begin{array}{r} {\scriptstyle 4\,1} \\ \cancel{5}2 \\ -\ 34 \\ \hline 8 \end{array}$$

Borrow 1 ten from tens. Then subtract 4 from 12.

Concrete Model



Take away 3 longs from 4 longs. (4 − 3 = 1 long)

So, 52 − 34 = 18.

Paper and Pencil

$$\begin{array}{r} {\scriptstyle 4\,1} \\ \cancel{5}2 \\ -\ 34 \\ \hline 18 \end{array}$$

Subtract 3 tens from 4 tens.

Regrouping

When a digit in the minuend is smaller than the digit below it, borrow from the next higher place to make it bigger. Records the regroupings above the minuend.

Example 1: Subtract 3356 from 8274

Borrow 1 thousand to make 12 hundreds. 7 thousands are left.

Borrow 1 ten to make 14 ones. 6 tens are left.

$$\begin{array}{r} \overset{7}{\cancel{8}}\,{}^{1}2\,\overset{6}{\cancel{7}}\,{}^{1}4 \\ -\;3\;3\;5\;6 \\ \hline 4\;9\;1\;8 \end{array}$$

← regrouping or borrowing numbers

Example 2: Subtract 335 from 704

Borrow 1 hundred to make 10 tens. 6 hundreds are left.

Borrow 1 ten to make 14 ones. 9 tens are left.

$$\begin{array}{r} \overset{6}{\cancel{7}}\,{}^{1}\overset{9}{\cancel{0}}\,{}^{1}4 \\ -\;3\;3\;5 \\ \hline 3\;6\;9 \end{array}$$

← regrouping or borrowing numbers

Checking Differences

Subtraction is checked by addition. Add the difference to the subtrahend. If this sum equals the minuend, the exact difference is correct.

Example: Subtract 59 from 1913 and check.

$$\begin{array}{r} 1\,\overset{8}{\cancel{9}}\,\overset{10}{\cancel{1}}\,{}^{1}3 \\ -\;\;\;5\;9 \\ \hline 1\;8\;5\;4 \end{array} \qquad \text{Check:} \qquad \begin{array}{r} 1\,{}^{1}8\,{}^{1}5\;4 \\ +\;\;\;5\;9 \\ \hline 1\;9\;1\;3 \end{array}$$

The exact difference is correct.

Note

- You may fill the missing digits in the subtrahend using zeros if you wish.

$$\begin{array}{r} 1913 \\ -\;0059 \\ \hline 1854 \end{array}$$ ← subtrahend

Practice Activities

Space for Your Work

Do **not** use a calculator for Questions 1 and 2.

1. Find the difference using paper and pencil methods.

a. $\begin{array}{r} 5985 \\ -\ 3780 \\ \hline \end{array}$

b. $\begin{array}{r} 4384 \\ -\ 1678 \\ \hline \end{array}$

c. $\begin{array}{r} 885\,312 \\ -\ 235\,209 \\ \hline \end{array}$

d. 2793 – 329

e. $\begin{array}{r} 50\,000 \\ -\ 6\,237 \\ \hline \end{array}$

f. $\begin{array}{r} 949\,888 \\ -\ 796\,902 \\ \hline \end{array}$

Space for Your Work

2. Fill in the missing digits.

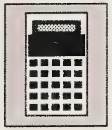
a.
$$\begin{array}{r} \square\,1 \\ -\ 3\,\square \\ \hline 4\,9 \end{array}$$

b.
$$\begin{array}{r} \square\,4\,6 \\ -\ 3\,\square\,5 \\ \hline 4\,4\,\square \end{array}$$

c.
$$\begin{array}{r} \square\,8\,5\,1\,\square \\ -\ 2\,\square\,\square\,6\,1 \\ \hline 3\,5\,4\,\square\,1 \end{array}$$

Use a calculator for Question 3.

Space for Your Work



3. The chart shows jam sales.

Jars of Jam Sold		
Kind	January	April
Strawberry	298	305
Raspberry	254	278

a. How many more jars of strawberry jam than raspberry jam were sold in January?

b. How many more jars of strawberry jam were sold in April than in January?

Computer Alternative

4. For further instruction and practice in subtraction, try the subtraction lesson, which is the second lesson, in the computer program *Conquering Whole Numbers* (MECC). Information on the program is in the folder with the disk.

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Twelve-year olds require about 10.4 megajoules or 10 400 kilojoules of food energy each day.

   Shawn is 12 years old. For dinner, he had the following.

Tomato juice	190 kJ	Broccoli	90 kJ
Roast beef	810 kJ	Strawberries	240 kJ
Baked potato ...	380 kJ	Milk, whole	660 kJ
Butter	450 kJ	2 slices bread .	600 kJ

[1]

   To have eaten the required amount for the day, how much food energy should have been provided by Shawn's other meals?

[1]Reproduced with permission of the Minister of Supply and Services Canada.

Space for Your Work

2. A pulp and paper company employed 15 003 people. There were 3362 loggers, 1604 office workers, and 5159 maintenance and construction workers. In addition, 772 people worked in the laboratories and 4106 workers processed the paper.

   a. How many of the people employed by the company are not loggers?

   b. How many more people work in the office than in the laboratories?

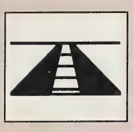
c. How many fewer people process the paper than work in maintenance and construction?

d. What is the difference of the numbers for the largest group of employees and the smallest group of employees?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

FINDING PRODUCTS



What Lies Ahead

In this section you will learn these skills.

- computing exact whole number products in column form
- checking the accuracy of exact whole number products

In this section you will use these words.

- multiplier
- multiplicand:
- product

Working Together

In this section you will practice your multiplication skills by using paper and pencil and by using a calculator.

Learning Aids Activities

You will begin by investigating multiplication using concrete models. You will need base 10 blocks for the Learning Aids Activities.

To prepare yourself for this section, turn to Exercise D in the *Learning Aids Booklet* and complete the activities.

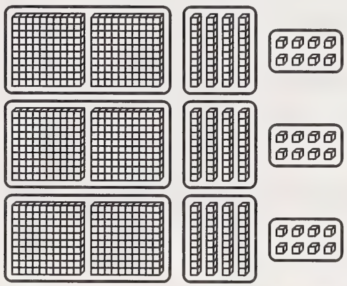
Working Together

Going from Concrete Models to Paper-and-Pencil Methods

Compare the steps in the paper and pencil methods with those using concrete models.

Example: 248×3

Concrete Model



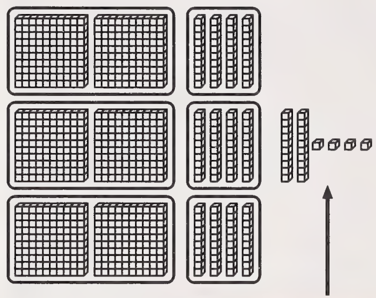
248 × 3 = 200 × 3 + 40 × 3 + 8 × 3

Form 3 groups of 200, 3 groups of 40 and 3 groups of 8.

Paper and Pencil

Think about the steps of multiplication.

Concrete Model



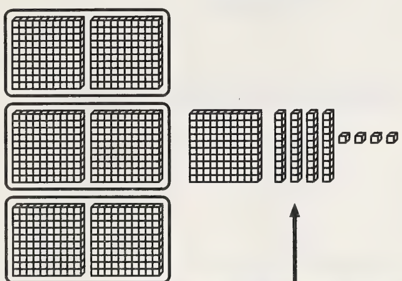
Begin with 3 groups of 8. Regroup the 24 units to 2 longs and 4 units.

Paper and Pencil

Multiply ones and regroup.

$$\begin{array}{r} {}^{2} \\ 248 \\ \times\ 3 \\ \hline 4 \end{array}$$

Concrete Model



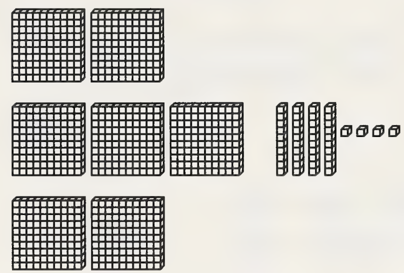
Now consider 3 groups of 4 longs. Regroup 12 longs for 1 flat and 2 longs.
(2 longs + 2 longs = 4 longs)

Paper and Pencil

Multiply tens, add the number carried, and regroup.

$$\begin{array}{r} {\scriptstyle 1\,2} \\ 248 \\ \times\ 3 \\ \hline 44 \end{array}$$

Concrete Model



Now consider 3 groups of 2 flats. Altogether now there are 7 flats, 4 longs and 4 units or 744.

Paper and Pencil

Multiply hundreds and add the number carried.

$$\begin{array}{r} {\scriptstyle 1\,2} \\ 248 \\ \times\ 3 \\ \hline 744 \end{array}$$

So, 248 × = 744.

One-Digit Multiplier

Multiply each digit in the multiplicand by the multiplier, going from right to left. Regroup when the column products are 10 or more.

Example 1

$$\begin{array}{r l} 83 & \leftarrow \text{multiplicand} \\ \times\ 3 & \leftarrow \text{multiplier} \\ \hline 249 & \end{array}$$

Example 2

$$\begin{array}{r l} {\scriptstyle 1\,3} & \leftarrow \text{regrouping or carrying numbers} \\ 237 & \\ \times\ 5 & \\ \hline 1185 & \end{array}$$

Note

- It is helpful to write the regrouping or carrying numbers above the multiplicand.

Two-Digit Multiplier

Multiply each digit in the multiplicand by each digit in the multiplier, going from right to left. Each multiplier digit makes a **partial product**. Add the partial products to find the final product.

Example

- Use zeros to maintain place value.

$$\begin{array}{r l} {\scriptstyle 5} & \\ {\scriptstyle 3} & \\ 59 & \\ \times\ 64 & \\ \hline 236 & \\ 3540 & \leftarrow \text{notice the zero} \\ \hline 3776 & \end{array}$$

- Or leave a space to maintain the place value.

$$\begin{array}{r l} {\scriptstyle 5} & \\ {\scriptstyle 3} & \\ 59 & \\ \times\ 64 & \\ \hline 236 & \\ 354 & \leftarrow \text{notice the space} \\ \hline 3776 & \end{array}$$

Three-Digit Multiplier

Multiply each digit in the multiplicand by each digit in the multiplier, going from right to left. Add the three partial products to find the final product.

Example

- Use zeros to maintain place value.

```
     1
     4
   406
 × 218
  3248
  4060  > notice the zeros
 81200
 88508
```

- Or leave spaces to maintain place value.

```
     1
     4
   406
 × 218
  3248
  406   > notice the spaces
 812
 88508
```

Zeros in the Multiplier

When multiplying by a 0 digit, you have a choice of methods.

- Write a line of 0's for the partial product.

Example

```
   32                 43
   64                 64
   175                175
 × 509              × 509
  1575       OR      1575
  0000               000
 87500              875
 89075              89075
```

- Use zeros to maintain place value and continue with the next multiplier digit.

Example

```
   43                 43
   64                 64
   175                175
 × 509              × 509
  1575       OR      1575
 87500              8750
 89075              89075
```

Practice Activities

Space for Your Work

Do **not** use a calculator for Questions 1 and 2.

1. Find the product using paper and pencil methods.

   a. $\begin{array}{r} 38 \\ \times\ 6 \\ \hline \end{array}$

   b. $\begin{array}{r} 17 \\ \times\ 49 \\ \hline \end{array}$

   c. $\begin{array}{r} 79 \\ \times\ 40 \\ \hline \end{array}$

   d. $\begin{array}{r} 105 \\ \times\ 38 \\ \hline \end{array}$

   e. $\begin{array}{r} 273 \\ \times\ 4 \\ \hline \end{array}$

   f. $\begin{array}{r} 196 \\ \times\ 303 \\ \hline \end{array}$

Space for Your Work

2. Complete.

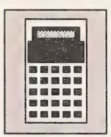
a.

$$\begin{array}{rrrrr} & & 4 & 0 & 8 \\ & & \times & 6 & 8 \\ \hline & 3 & \square & \square & 4 \\ \square & \square & 4 & 8 & \\ \hline \square & 7 & \square & 4 & 4 \end{array}$$

b.

$$\begin{array}{rrrr} & & 9 & 3 \\ & \times & \square & 6 \\ \hline & \square & 5 & \square \\ \square & 7 & 9 & \\ \hline \square & 3 & \square & 8 \end{array}$$

Use a calculator in Questions 3-5.



3. One box of potato chips holds 200 g. There are 24 boxes packed in a case. How many grams of potato chips are there in one case?

Space for Your Work

4. A skateboard company has orders for 10 000 skateboards to be shipped by the middle of November. If there are 98 working days left before the delivery date, and if 105 skateboards are made per day, will there be enough skateboards ready to fill the order?

5. A cafeteria sells 294 hamburgers a day. There are 250 working days in a year. How many hamburgers are sold in a year?

Computer Alternative

6. If you wish to have more instruction and practice in multiplication, try the program for multiplication in *Conquering Whole Numbers* (MECC). Information about the program is in the folder with the disk.

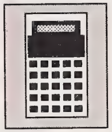
Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Use your calculator for these activities



1. Find your age in hours. Use your age as of your last birthday.

2. How many days are there in a decade?

3. Many calculators will only display 8 digits. The largest number that can be shown is 99 999 999. Overflow occurs when a result is greater.

Key Press	Display
9 0 0 0 0 0 0 0 × 2 0 =	E 1.8000000000

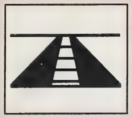
Find the products using paper-and-pencil methods. Then predict the calculator display.

a. 600 000 × 600

b. 15 000 000 × 20

See your learning facilitator to check your answers and to receive further instructions.

FINDING QUOTIENTS



What Lies Ahead

In this section you will review these skills.

- computing exact whole number quotients
- checking the accuracy of whole number quotients

In this section you will use these words.

- dividend
- divisor
- quotient
- remainder

Working Together

Now it is time to get out the paper and pencil and to tackle that dreaded long division. Actually, it's not so bad!

Learning Aids Activities

You will begin by investigating division using concrete models. You will need base 10 blocks.

To prepare yourself for this section, turn to Exercise E in the *Learning Aids Booklet* and complete the activities.

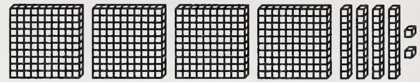
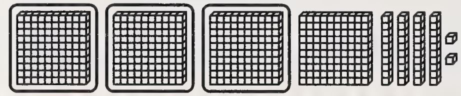
Working Together

Going from Concrete Models to Paper-and-Pencil Methods

As you have learned from the learning aids activities, you can model quotients with base 10 blocks.

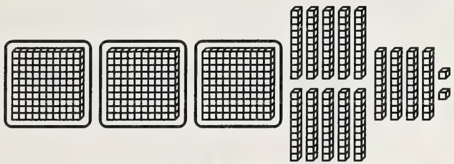
Compare the steps in the paper and pencil method with the steps in the concrete model.

Example: $442 \div 3$

Concrete Model	Paper and Pencil
 Form 442.	
 $442 \div 3$ means 3 groups of how many? Begin with 4 flats and divide them into 3 groups of 1. There is 1 flat left over.	Divide hundreds and subtract. $$\begin{array}{r} 1 \\ 3\overline{)442} \\ -3 \\ \hline 1 \end{array}$$

Concrete Model	Paper and Pencil

Concrete Model

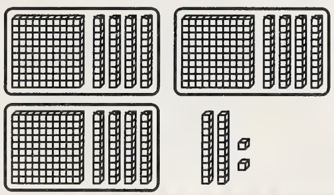


Trade 1 flat for 10 longs. There are now 14 longs.

Paper and Pencil

Regroup 1 hundred for 10 tens.

$$\begin{array}{r} 1 \\ 3\overline{)442} \\ -3 \\ \hline 14 \end{array}$$

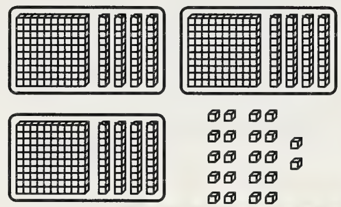


Now divide 14 longs into 3 groups of 4. There are 2 longs left over.

Divide tens and subtract.

$$\begin{array}{r} 14 \\ 3\overline{)442} \\ -3 \\ \hline 14 \\ -12 \\ \hline 2 \end{array}$$

Concrete Model

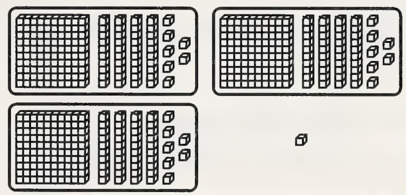


Trade 2 longs for 20 units. There are now 22 units.

Paper and Pencil

Regroup 2 tens for 20 ones.

$$\begin{array}{r} 14 \\ 3\overline{)442} \\ -3 \\ \hline 14 \\ -12 \\ \hline 22 \end{array}$$



Finally, divide 22 units into 3 groups of 7. There is 1 unit left over.

Divide ones and subtract.

$$\begin{array}{r} 147 \\ 3\overline{)442} \\ -3 \\ \hline 14 \\ -12 \\ \hline 22 \\ -21 \\ \hline 1 \end{array}$$

So, $442 \div 3 = 147$ R1.

Rules for Dividing

Now that you understand why the paper and pencil method of division works, review the steps and consider these examples.

Step 1: Choose the fewest possible number of dividend digits that can be divided by the divisor.

Step 2: Divide the dividend digits by the divisor to obtain a quotient digit.

Step 3: Multiply the quotient digit by the divisor.

Step 4: Subtract the product from the dividend digits.

Step 5: Bring down the next dividend digit to form the next group of dividend digits to be divided.

Step 6: Continue the cycle until all the dividend digits are used.

Examples

```
                 65   ← quotient
divisor →  7 ) 455    ← dividend
               42↓
               ---
                35
                35
                --
                 0
```

```
                 54   ← quotient
divisor → 83 ) 4509
               415↓
               ----
                359
                332
                ---
                 27   ← remainder
```

```
                107   ← quotient
divisor → 39 ) 4180
               39↓↓
               ----
                280
                273
                ---
                  7   ← remainder
```

Note

- Sometimes there will be a remainder.
- In the last example the zero is used as a place holder in the quotient.
- For every number you bring down from the dividend, you must have a number in the quotient.

Using Guess-Check-Revise Methods

Some guessing and testing may be necessary when you are working with large numbers.

Example: 54 422 ÷ 214

Trial 1

```
       3
214)58422
    642
```

← 642 is larger than 584 so try a smaller quotient digit.

Trial 2

```
      26
214)58422
    428↓
    1562
    1284
     278
```

← The remainder is larger than the divisor so use a larger quotient digit.

Trial 3

```
     273
214)58422
    428↓
    1562
    1498↓
     642
     642
       0
```

Short Division

Short division is long division without all the writing! In short division. The multiplication and subtraction are done mentally. The remainders are written as subscripts.

Examples

Short Division	Long Division
$4\overline{)2\,5_1 2}$ = 6 3 R0	$4\overline{)252}$ = 63 R0
$8\overline{)1\,1_3 6_4 3}$ = 1 4 5 R3	$8\overline{)1163}$ = 145 R3

```
  63 R0
4)252
  24
   12
   12
```

```
  145 R3
8)1163
  8
  36
  32
   43
   40
    3
```

Note

Short division is rather difficult when the divisors have more than one digit.

Practice Activities

Space for Your Work

Do **not** use calculators for Questions 1 and 2.

1. Find the quotient using long division.

   a. $4\overline{)2248}$

   b. $7\overline{)15821}$

   c. $12\overline{)4308}$

   d. $73\overline{)9858}$

   e. $352\overline{)9856}$

Space for Your Work

2. Find the quotient using short division.

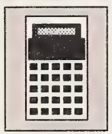
   a. $8\overline{)3046}$

   b. $4\overline{)3662}$

   c. $3\overline{)1905}$

   d. $3\overline{)5436}$

Use a calculator for Questions 3-5.



3. Find the quotient.

   a. $6381 \div 9$

   b. $46\overline{)66\,930}$

   c. $\dfrac{43\,010\,000}{850}$

Space for Your Work

4. A figure skating club sold 98 tickets for a total of \$294. What was the price of one ticket?

5. A drama theatre can seat 1036 people. There are 28 rows, each with the same number of seats. How many seats are there in each row?

Computer Alternative

Space for Your Work

6. The computer program *Conquering Whole Numbers* (MECC) has a lesson on division. If you wish to have more instruction and practice, use that program. Instructions are given with the disk.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Checking Division

You have learned to check division by multiplying.

Example

```
       115 R41
 49 ) 5676
      49
       77
       49
       286
       245
        41
```

Check

```
  115  ←quotient
×  49  ←divisor
 1035
 460
 5635
+  41  ←remainder
 5676  ←dividend
```

Casting Out Nines

Here is another method to check division mentally. It is called **casting out nines**.

Step 1: Add digits in the divisor, quotient, remainder, and dividend until each is a single digit.

quotient digit
$1 + 1 + 5 = 7$

divisor digit
$4 + 9 = 13$
$1 + 3 = 4$

```
     115 R41
49 ) 5676
```

remainder digit
$4 + 1 = 5$

dividend digit
$5 + 6 + 7 + 6 = 24$
$2 + 4 = 6$

Step 2: Multiply the divisor digit and the quotient digit. Then add the remainder digit. Finally, add the resulting digits until there is a single-digit number.

$$\begin{array}{r l} 7 & \leftarrow \text{quotient digit} \\ \times\ 4 & \leftarrow \text{divisor digit} \\ \hline 28 & \\ +\ 5 & \leftarrow \text{remainder digit} \\ \hline 33 & \leftarrow \text{resulting digits} \end{array}$$

$3 + 3 = 6$ ←resulting single-digit number

Step 3: Compare the resulting single-digit number and the dividend digit.

If this single-digit number is the same as the dividend digit, the division is probably correct.

The dividend digit in this example is 6.
The resulting single-digit number is 6.

So $49\overline{)5676}$ with quotient 115 R41 is probably correct.

Concluding Activities

Space for Your Work

Use the casting-out-nines method to check the division answers of the Practice Activities.

1. a. $4\overline{)2248}$

   b. $7\overline{)15821}$

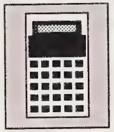
Space for Your Work

c. $12\overline{)4308}$

d. $73\overline{)9858}$

e. $352\overline{)9856}$

Space for Your Work



2. Use a calculator and the guess-check-revise method of problem solving to figure out where to place a division sign between the numerals to make a true statement. One has been done as an example.

   a. 48024 = 20

   b. 3046432 = 952

   c. 1728144 = 12

   d. 17289 = 192

3. Each of these questions needs 2 division signs. One has been done as an example.

   a. 384234 = 8

   b. 12501025 = 5

2. a. $480 \div 24 = 20$

3. a. $(384 \div 2) \div 24 = 8$

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this summary you will review the skills you learned in Part One.

- reading and writing whole numbers
- comparing and ordering whole numbers
- rounding whole numbers
- adding, subtracting, multiplying, and dividing numbers by using paper and pencil, and by using a calculator

Working Together

At this point, it is a good idea to review the skills you have learned in Part One.

Turn to Section 1 and review the Pretest. Correct any errors you may have made. You may be pleasantly surprised to discover how much you have learned.

PART TWO

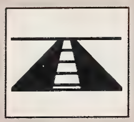
WESTFILE, INC.

Part One dealt with computation using paper and pencil or using a calculator.

Part Two deals with mental computation. Some people have extraordinary powers of mental computation. For example, before Zerah Colburn (1804-1840) was 10 years old, he could instantly multiply mentally numbers like 21 734 and 543. You probably aren't as talented as Zerah Colburn, but you can improve your ability to compute mentally with instruction and practice.

Part Two also deals with order of operations. When you get dressed you put on your socks before you put on your shoes. When you calculate a series of operations, you also follow a certain order. You will learn about the rules for order of operations in this part of the module.

GETTING SET



What Lies Ahead

This section will test these skills.

- computing mentally exact whole number sums, differences, products, and quotients
- using the rules for the order of operations

Working Together

The pretest in this section will help you and your learning facilitator discover your strengths and weaknesses.

Pretest

Space for Your Work

1. Evaluate mentally.

   a. $28 + 15 + 2$

   b. $13 + 15 + 5 + 7$

   c. $45 + 38$

   d. $73 + 26$

   e. $83 - 69$

Space for Your Work

f. $72 - 53$

g. 99×14

h. $(15 \times 2) + (15 \times 8)$

i. $20\,000 \div 400$

j. $380 \div 4$

Space for Your Work

2. Evaluate. Show all steps in the space provided.

a. $29 - 5 \times 3$

b. $56 \div (10 + 6 - 8)$

c. $7 + 7 \times 7 + 7$

d. $\dfrac{3 \times 5 - 1}{16 \div 8}$

See your learning facilitator to check your answers and to receive further instructions.

ADDING MENTALLY

What Lies Ahead

In this section you will learn this skill.

- computing exact whole number sums mentally

In this section you will use these words.

- commutative property
- associative property
- identity element

Working Together

It is time to warm up the calculator in your head! This section will help you to add whole numbers mentally.

Video Activity

View the video *MATHWORKS: Using Mental Computation for Addition* (AIT). Then read the strategies for mental addition in this section.

If you cannot view the video, read the strategies for mental addition.

There are three properties that will help you add mentally — the identity element, the commutative property, and the associative property.

The names of these properties may be easier to remember and understand if you relate them to words you know. For example these words may help.

- **Identical** twins look the same.
- When a person **commutes**, the distance from home to work equals the distance from work to home.
- When a boy is **associated** with Scouts, he belongs to that group.

The Additive Identity

Adding zero to any number does not change the sum. The sum remains the same.

Example 1

$$9 + 0 = 9$$
$$9 = 9$$

Example 2

$$0 + 356 = 356$$
$$356 = 356$$

This means that you can ignore zeros when you add strings of numbers.

Commutative Property of Addition

Numbers can be added in any order without changing the sum.

Example 1

$$6 + 8 = 8 + 6$$
$$14 = 14$$

Example 2

$$75 + 430 = 430 + 75$$
$$505 = 505$$

Associative Property of Addition

Numbers can be grouped in different ways for addition without changing the sum.

Example 1

$$(3 + 5) + 5 = 3 + (5 + 5)$$

Numbers in brackets are added first.

$$8 + 5 = 3 + 10$$
$$13 = 13$$

Example 2

$$(247 + 50) + 150 = 247 + (50 + 150)$$
$$297 + 150 = 247 + 200$$
$$447 = 447$$

This property allows you to add strings of numbers in the order which is easiest.

There are three strategies that will help you add mentally — the left-right method, the plus-minus method, and the friendly-numbers method.

The Left-Right Method

You can add numbers mentally by breaking them up and adding from left to right.

Example 1

$$\begin{array}{r} 73 \\ +\ 26 \\ \hline \end{array} \rightarrow \begin{array}{r} 70 + 3 \\ 20 + 6 \\ \hline 90 + 9 \end{array} = 99$$

Example 2

$$\begin{aligned} & 132 + 256 \\ &= (100 + 200) + (30 + 50) + (2 + 6) \\ &= 300 + 80 + 8 \\ &= 388 \end{aligned}$$

Note

This method is best used in addition problems where regrouping is not ordinarily required.

$$\begin{array}{r} 73 \\ +\ 26 \\ \hline 99 \end{array} \qquad \begin{array}{r} 132 \\ +\ 256 \\ \hline 388 \end{array}$$

The Plus-Minus Method

You can add numbers mentally by rounding one of the numbers and adjusting the other number to maintain the equality.

Example 1

$$\begin{array}{r} 45 \\ +\ 38 \\ \hline \end{array} \rightarrow \begin{array}{r} 45 - 2 \\ +\ 38 + 2 \\ \hline \end{array} \rightarrow \begin{array}{r} 43 \\ +\ 40 \\ \hline 83 \end{array}$$

Round 38 to 40. Because you added 2 to 38, take away 2 from 45.

Example 2

$$\begin{aligned} & 64 + 237 \\ &= 61 + 240 \\ &= 301 \end{aligned}$$

Round 237 to 240. Because you added 3 to 237, take away 3 from 64.

Note

This method is best used in addition problems where regrouping is ordinarily required.

$$\begin{array}{r} {}^{1} \\ 45 \\ +\ 38 \\ \hline 83 \end{array} \qquad \begin{array}{r} {}^{1} \\ 64 \\ +\ 237 \\ \hline 301 \end{array}$$

Friendly Numbers

When adding strings of numbers, you can look for combinations of digits which add up to 10, 20, 30, 40, 50, etc.

Example 1

$$\begin{array}{r} 3 \\ 4 \\ +\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 10 \\ +\ 4 \\ \hline 14 \end{array}$$

Example 2

$$\begin{aligned} & 5 + 1 + 3 + 6 + 2 \\ &= 20 + 1 + 6 \\ &= 27 \end{aligned}$$

When adding strings of larger numbers, you can look for combinations of numbers which add up to 100, 200, 300, 400, 500, etc.

Example 1

$$\begin{array}{r} 50 \\ 26 \\ +\ 50 \\ \hline \end{array} \qquad \begin{array}{r} 100 \\ +\ 26 \\ \hline 126 \end{array}$$

Example 2

$$\begin{aligned} & 248 + 20 + 152 + 180 \\ &= 400 + 200 \\ &= 600 \end{aligned}$$

Practice Activities

Space for Your Work

1. Are the following equations true or false? Why?

   a. $1803 + 0 = 0$

   b. $(69 + 33) + 7 = 69 + (33 + 7)$

   c. $12 + 7 + 8 + 13 = (7 + 8) + 10 + 13$

   d. $800 + 5 + 60 + 1200$
   $= (1200 + 800) + (60 + 5)$

Space for Your Work

2. Add mentally. Look for sums of 10.

   a. $7 + 6 + 8 + 2 + 3$

   b. $6 + 9 + 1 + 4 + 8$

   c. $3 + 2 + 2 + 8 + 7 + 5 + 4 + 5$

   d. $7 + 8 + 1 + 2 + 9 + 3 + 5 + 8 + 5$

Space for Your Work

3. Add mentally. Look for sums of 100.

   a. 75 + 60 + 29 + 25 + 40

   b. 50 + 1 + 24 + 76 + 50 + 99 + 3

   c. 25 + 60 + 25 + 40 + 50 + 18

4. Find the sum mentally. Look for possible changes in order or grouping.

   a. 19 + 15 + 1

   b. 127 + 44 + 3

   c. 159 + 13 + 7 + 21

Space for Your Work

5. Add mentally by the left-right method.

   a. 27 + 42

   b. $$\begin{array}{r} 12 \\ 61 \\ +\ 215 \\ \hline \end{array}$$

   c. 23 + 66

6. Find the sum mentally by the plus-minus method.

   a. 33 + 57

   b. $$\begin{array}{r} 24 \\ +\ 78 \\ \hline \end{array}$$

   c. 55 + 138

Space for Your Work

7. Add mentally by a method of your choice.

a. $65 + 3 + 15 + 6 + 5 + 1$

b.
$$\begin{array}{r} 81 \\ +\ 93 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 176 \\ +\ 495 \\ \hline \end{array}$$

d. $143 + 71 + 19$

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Find the sums mentally. Use any of the strategies you learned in this lesson.

   a. $5 + 3 + 10 + 14 + 2 + 6$

   b. $\begin{array}{r} 35 \\ +\ 62 \\ \hline \end{array}$

   c. $\begin{array}{r} 29 \\ +\ 46 \\ \hline \end{array}$

   d. $\begin{array}{r} 252 \\ +\ 347 \\ \hline \end{array}$

   e. $50 + 27 + 18 + 100 + 12$

Space for Your Work

2. Mentally compute the distance from Vancouver to Halifax.

Road Distances (km)	
Vancouver to Toronto	4492
Toronto to Montreal	539
Montreal to Halifax	1318

[1]

See your learning facilitator to check your answers and to receive further instructions.

[1]Statistics Canada

Working Together

Cross Out Singles is a fun mental addition game for two or more people.

Rules for Cross Out Singles

Step 1: The die is rolled nine times. As the numbers are called, the players write the numbers anywhere on their round-1 charts. Once written, the numbers cannot be moved.

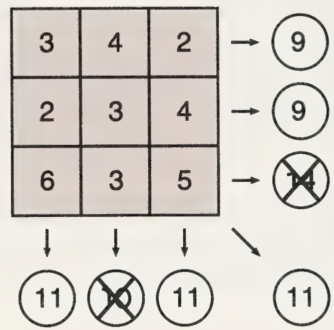
Step 2: Players then find the sums of the three rows, the three columns, and the diagonal from left to right. The sums are then recorded in the circles as shown.

Step 3: The players check their own sums. Any sum that appears in only one circle must be crossed out. This why the game is called "Cross Out Singles."

The total of the sums that are not crossed out is that player's score for the round.

See the example at the right.

Example



9 + 9 + 11 + 11 + 11 = 51

Score = 51

Step 4: After three rounds, the three scores are added. The player with the highest total for the three rounds is the winner.

Concluding Activities

Play Cross Out Singles[1]. You will find the score sheets in the appendix of the module booklet.

See your learning facilitator to discuss your game results.

[1]National Council of Teachers of Mathematics for game from *Ideas from the Arithmetic Teacher*, Reston, Virginia, 1979

SUBTRACTING MENTALLY

What Lies Ahead

In this section you will learn this skill.

- computing exact whole number differences mentally

In this section you will use these words.

- subtrahend
- difference
- minuend

Working Together

In this section you will do mental subtraction. You will use two strategies. These strategies are the left-right method and the attention method.

Video Activity

Your first activity in this section will be to view a video on the fine art of subtracting mentally. Note carefully the two strategies which are explained.

View the video *MATHWORKS: Using Mental Computation for Subtraction* (AIT). Then study the notes on mental subtraction in this section.

If you cannot view the video, study the following examples which show mental subtraction.

The Left-Right Method

You can subtract numbers by breaking them up and subtracting from left to right.

Example 1

$$\begin{array}{r} 287 \\ -\ 145 \\ \hline \end{array} \rightarrow \begin{array}{ccccccc} 200 & + & 80 & + & 7 & & \\ 100 & + & 40 & + & 5 & & \\ \hline 100 & + & 40 & + & 2 & + & 142 \end{array}$$

Example 2

$$\begin{aligned} & 395 - 172 \\ &= (300 - 100) + (90 - 70) + (5 - 2) \\ &= 200 + 20 + 3 \\ &= 223 \end{aligned}$$

Note

- This method is best used in subtraction problems where regrouping is not ordinarily needed.

$$\begin{array}{r} 287 \\ -\ 145 \\ \hline 142 \end{array}$$

The Attention Method

You can round the subtrahend and adjust the minuend to maintain the equality.

Example 1

$$\begin{array}{r} 171 \\ -\ 62 \\ \hline \end{array} \rightarrow \begin{array}{r} 171 - 2 \\ -\ \ 62 - 2 \\ \hline \end{array} \rightarrow \begin{array}{r} 169 \\ -\ 60 \\ \hline 109 \end{array}$$

Round 62 to 60. Because you took 2 from 62, also take 2 from 171.

Example 2

$$\begin{aligned} & 182 - 39 \\ &= 183 - 40 \\ &= 143 \end{aligned}$$

Round 39 to 40. Because you added 1 to 39, also add 1 to 183.

Note

- If you subtract from the subtrahend, subtract from the minuend. If you add to the subtrahend, add to the minuend.
- This method is best used in subtraction problems where regrouping is ordinarily needed.

$$\begin{array}{r} {\scriptstyle 6\,1} \\ 1\cancel{7}1 \\ -\ \ 62 \\ \hline 109 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 1\cancel{8}2 \\ -\ \ 37 \\ \hline 143 \end{array}$$

Practice Activities

Space for Your Work

1. What do you get when you subtract zero from a number? Write an example.

2. Is there a commutative property for subtraction? That is, can you reverse the order of the numbers? Write an example to support your answer.

3. Subtract mentally by the left-right method.

   a. $\begin{array}{r} 36 \\ -\ 13 \\ \hline \end{array}$

   b. $\begin{array}{r} 147 \\ -\ 25 \\ \hline \end{array}$

   c. $456 - 423$

Space for Your Work

4. Find the difference mentally by using the attention method.

   a. $63 - 19$

   b. $\begin{array}{r} 615 \\ -\ 498 \\ \hline \end{array}$

   c. $\begin{array}{r} 106 \\ -\ 66 \\ \hline \end{array}$

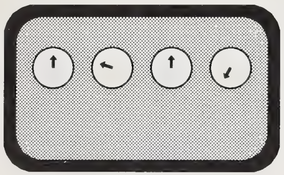
5. Calculate the following mentally. Use a strategy of your choice.

   a. $\begin{array}{r} 80 \\ -\ 44 \\ \hline \end{array}$

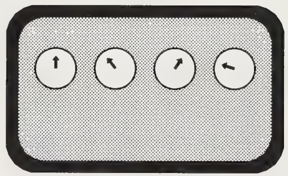
   b. $321 - 123$

   c. $\begin{array}{r} 3059 \\ -\ 1046 \\ \hline \end{array}$

6. In January, a gas meter read 0806. In February it read 0918. Compute mentally the increase in the meter reading.



January



February

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Subtract the following mentally. Use any of the strategies you learned in this section.

   a. $99 - 25$

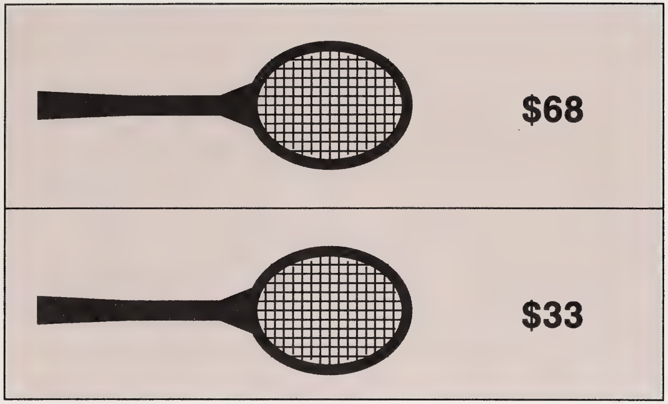
   b. $\begin{array}{r} 70 \\ -\ 39 \\ \hline \end{array}$

   c. $335 - 124$

   d. $\begin{array}{r} 112 \\ -\ 83 \\ \hline \end{array}$

   e. $60 - 20$

2. Mentally find the difference in the costs of these tennis rackets.



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

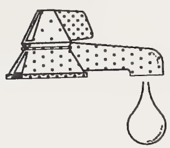
Concluding Activities

Space for Your Work

1. The plus-minus method and the attention method are similar. Explain the difference between these methods. Give examples.

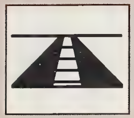
2. Try this problem mentally.

   Mr. Chase's water meter presently reads 8 530 000 L. Six months ago it read 8 185 000 L. How much water did the Chase family use in this six-month period?



See your learning facilitator to check your answers and to receive further instructions.

MULTIPLYING MENTALLY



What Lies Ahead

In this section you will learn this skill.

- computing exact whole number products mentally

In this section you will use these words.

- commutative property
- associative property
- distributive property
- identity element
- zero property

Working Together

Do you want to be a multiplication monster? This section will help you.

Video Activity

View the video, *SOLVE IT: Using Mental Computation for Multiplication* (AIT) to see the man who was once the "fastest mental multiplier in the whole west"! Then read the notes in this section.

If you cannot view the video, study the examples of the strategies for mental multiplication on the following pages.

Multiplication Properties

In this section you will review several useful multiplication properties. Some of these look quite a bit like those for addition.

Remember it is helpful to relate math words to similar words you already know. Can you think of a way to relate the "distributive property"?

One way is this:

> If you were to **distribute** candies to a group of children you would give each child the same number of candies.

The Zero Property

The product of any number and 0 is 0.

Example

$$8 \times 0 = 0$$
$$0 = 0$$

The Multiplication Identity

The product of any number and 1 is the number itself.

Example

$$5 \times 1 = 5$$
$$5 = 5$$

Commutative Property of Multiplication

Changing the order of the factors does not change the product.

Example 1

$$4 \times 52 = 52 \times 4$$
$$208 = 208$$

Example 2

$$22 \times 3 = 3 \times 22$$
$$66 = 66$$

Associative Property of Multiplication

Grouping factors in different ways does not change the product.

Example 1

$$5 \times (20 \times 4) = (5 \times 20) \times 4$$
$$5 \times 80 = 100 \times 4$$
$$400 = 400$$

Numbers in brackets are dealt with first.

Example 2

$$25 \times (3 \times 4) = (25 \times 4) \times 3$$
$$25 \times 12 = 100 \times 3$$
$$300 = 300$$

This method allows you to multiply strings of numbers in the order that is most convenient and to look for friendly numbers. Friendly numbers are numbers that are easy to work with.

Distributive Property of Multiplication Over Addition

The same result is produced whether you multiply a factor by a sum or by individual addends.

Example

$$\begin{aligned} 5 \times (10 + 5) &= 5 \times 15 \\ &= 75 \end{aligned}$$

OR

$$\begin{aligned} 5 \times (10 + 5) &= 5 \times 10 + 5 \times 5 \\ &= 50 + 25 \\ &= 75 \end{aligned}$$

This property allows you to use the left-right method which is explained on the next page.

Distributive Property of Multiplication Over Subtraction

The same result is achieved whether you mutliply a factor by a difference or by the minuend and subtrahend individually.

Example

$$\begin{aligned} 5 \times (10 - 5) &= 5 \times 5 \\ &= 25 \end{aligned}$$

OR

$$\begin{aligned} 5 \times (10 - 5) &= 5 \times 10 - 5 \times 5 \\ &= 50 - 25 \\ &= 25 \end{aligned}$$

This property allows you to use the plus-minus method described on the next page.

There are 4 strategies which will help you multiply mentally. These strategies are the left-right method, the plus-minus method, special products, and friendly numbers.

The Left-Right Method

You can multiply mentally by breaking the larger factor and multiplying from left to right.

Example 1

$$
\begin{aligned}
& 8 \times 145 \\
&= (8 \times 100) + (8 \times 40) + (8 \times 5) \\
&= 800 + 320 + 40 \\
&= 1160
\end{aligned}
$$

Example 2

$$
\begin{aligned}
& 267 \times 3 \\
&= (200 \times 3) + (60 \times 3) + (7 \times 3) \\
&= 600 + 180 + 21 \\
&= 801
\end{aligned}
$$

The Plus-Minus Method

You can round the larger factor and adjust to balance the rounding.

Example 1

Add 2 to 68. When 70 is multiplied by 7, 2 sevens are added.

Subtract 2 sevens to match the 2 sevens added.

$$
\begin{aligned}
68 \times 7 &= (70 \times 7) - (2 \times 7) \\
&= 490 - 14 \\
&= 476
\end{aligned}
$$

Example 2

Subtract 3 from 83.

When 80 is multiplied by 4, 4 threes are subtracted. Add 4 threes to match the 4 threes subtracted.

$$
\begin{aligned}
83 \times 4 &= (80 \times 4) + (3 \times 4) \\
&= 320 + 12 \\
&= 332
\end{aligned}
$$

Special Products

You can multiply a whole number by 10, 100, or 1000, just by adding 1, 2, or 3 zeros to the whole number.

Example 1 $182 \times 10 = 182 + 1$ zero
$= 1820$

Example 2 $73 \times 100 = 73 + 2$ zeros
$= 7300$

Example 3 $19 \times 1000 = 19 + 3$ zeros
$= 19\,000$

When one or both factors end in one zero or several zeros, you can first multiply the front digits and then add the total number of zeros contained in the factors used in the problem.

Example 1 $9 \times 600 = (9 \times 6) + 2$ zeros
$= 54 + 2$ zeros
$= 5400$

Example 2 $200 \times 7000 = (2 \times 7) + 5$ zeros
$= 14 + 5$ zeros
$= 1\,400\,000$

Friendly Numbers

When multiplying strings of numbers, you can look for factors that are easy to work with.

Example 1 $25 \times 3 \times 4 = 100 \times 3$
$= 300$

Example 2 $3 \times \boxed{8 \times 5} = 3 \times 40$
$= 120$

Practice Activities

Space for Your Work

Do either Question 1 or 2.

1. If you watched the video, answer the following questions about the video *SOLVE IT: Using Mental Computation for Multiplication*.

   a. What did Jimmy and Claude have to do when the power in the store went off?

   b. Describe the two methods of mental multiplication used by Great-great-granddaddy Claude.

2. If you did not watch the video, explain when in everyday life you might need to multiply mentally.

3. True or false?

   a. $12 \times 0 = 12$

   b. $1 \times 129 = 129$

   c. $9 \times 88 = 88 \times 9$

   d. $3 \times (8 + 20) = 3 \times 8 + 20$

Space for Your Work

4. Use = which means is equal to or $\neq$ which means is not equal to in each of the following to make a true statement.

   a. $69 \times 1 \bigcirc 69 \times 0$

   b. $(8 \times 5) \times 2 \bigcirc 8 \times (5 \times 2)$

   c. $(6 \times 9) + (4 \times 9) \bigcirc (6 + 4) \times 9$

   d. $(24 \times 1) + (24 \times 3) \bigcirc 24 \times (1 + 3)$

   e. $(16 \times 3) - (14 \times 3) \bigcirc (16 - 4) \times 3$

   f. $(20 + 7) \times (20 + 3) \bigcirc 20 + (7 \times 3)$

5. Multiply mentally.

   a. 1000×55

   b. 4×90

   c. 20×50

   d. 1200×300

   e. $80\,000 \times 700$

Space for Your Work

6. Find the product mentally.

   a. $5 \times 26 \times 2$

   b. $15 \times 6 \times 5$

   c. $25 \times 18 \times 4$

   d. $12 \times 4 \times 5$

   e. $20 \times 27 \times 5$

   f. $38 \times 25 \times 4$

   g. $(16 \times 2) + (16 \times 18)$

   h. 12×505

7. Multiply mentally by the plus-minus method.

   a. 51×9

   b. 6×86

   c. 98×4

Space for Your Work

8. Calculate mentally by the left-right method.

   a. 123×4

   b. 7×609

   c. 555×8

9. Find the product mentally using a method of your choice.

   a. 67×5

   b. 7×249

   c. 102×9

   d. 42×30

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Multiply mentally.

   a. $9 \times 4 \times 5$

   b. $12 \times 5 \times 7$

   c. $25 \times 20 \times 4$

   d. 4×80

   e. 900×300

2. Multiply mentally. Use the strategies you learned in this section.

   a. 49×6

   b. 2×135

   c. 72×5

See your learning facilitator to check your answers and to receive further instructions.

Cross Out Singles[1] is a fun mental multiplication game for two or more people.

Rules for Cross Out Singles

Step 1: The 2 dice are rolled nine times. Each time the two numbers are called, the players calculate the product of the 2 numbers and write the product anywhere on their round-1 charts. Once written, the numbers cannot be moved.

Step 2: Players then find the sum of three rows, the three columns, and the diagonal, and record them in the circles.

Step 3: Players check their own sums. Any sum that appears in only one circle must be crossed out.

The total of the sums that are not crossed out is the player's score for the round.

See the example at the right.

Example

36	6	15	→ ~~57~~
10	6	15	→ ~~31~~
4	24	8	→ 36
↓ 50	↓ 36	↓ ~~33~~	↘ 50

$(2 \times 36) + (2 \times 50) = 172$

Score = 172

Step 4: After three rounds, the three scores are added. The player with the highest total score wins.

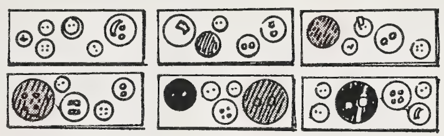
[1] National Council of Teachers of Mathematics for game from *Ideas from the Arithmetic Teacher*, Reston, Virginia, 1979

Concluding Activities

Space for Your Work

1. Solve mentally.

   a. Lucy has 12 cards each containing 8 smaller buttons, and 12 cards each containing 5 large buttons. In all how many buttons does Lucy have?



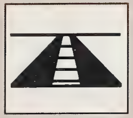
   b. One crate has 36 1-L pails of maple syrup. How many litres of maple syrup are there in 30 crates?

   c. A box of cookies has 2 rows of cookies. Each row has 11 cookies. There are 12 boxes in a crate. How many cookies are in the crate?

2. Play Cross Out Singles.The score sheets are in the appendix of the Module Booklet.

See your learning facilitator to check your answers and discuss your game results.

DIVIDING MENTALLY



What Lies Ahead

In this section you will learn this skill.

- computing exact whole number quotients mentally

In this section you will use these words.

- quotient
- dividend
- divisor
- distributive property
- identity element
- undefined

Working Together

It is time to polish up your division skills. This section will help you divide mentally.

Division questions can be written in three different ways.

Example: Divide 64 by 8.

- First Way

$$\underset{\text{dividend}}{\underset{\uparrow}{64}} \div \underset{\text{divisor}}{\underset{\uparrow}{8}} = \underset{\text{quotient}}{\underset{\uparrow}{8}}$$

- Second Way

$$\begin{array}{rl} 8 & \leftarrow \text{quotient} \\ \underset{\text{divisor}}{\underset{\uparrow}{8}}\overline{)64} & \leftarrow \text{dividend} \end{array}$$

- Third Way

$$\frac{\text{dividend} \rightarrow 64}{\text{divisor} \rightarrow 8} = 8 \leftarrow \text{quotient}$$

The following division properties will help you divide mentally.

Distributive Property of Division Over Addition

The same result is produced if you divide a sum or divide the addends individually.

Example

$$\begin{aligned}(16 + 8) \div 4 &= 24 \div 4 \\ &= 6\end{aligned}$$

OR

$$\begin{aligned}(16 + 8) \div 4 &= (16 \div 4) + (8 \div 4) \\ &= 4 + 2 \\ &= 6\end{aligned}$$

Distributive Property of Division Over Subtraction

The same result is produced if you divide a difference or divide the minuend and subtrahend individually.

Example

$$\begin{aligned}(16 - 8) \div 4 &= 8 \div 4 \\ &= 2\end{aligned}$$

OR

$$\begin{aligned}(16 - 8) \div 4 &= (16 \div 4) - (8 \div 4) \\ &= 4 - 2 \\ &= 2\end{aligned}$$

There are 3 strategies to help you divide mentally — the left-right method, the plus-minus method, and the special products method.

Left-Right Method

When you divide mentally you can sometimes split the dividend into parts which can be easily divided by the divisor. These partial quotients are then added to get the final answer.

Example 1

$$\begin{aligned}215 \div 5 &= (200 \div 5) + (15 \div 5)\\ &= 40 + 3\\ &= 43\end{aligned}$$

Example 2

$$\begin{aligned}147 \div 7 &= (140 \div 7) + (7 \div 7)\\ &= 20 + 1\\ &= 21\end{aligned}$$

Plus-Minus Method

When you divide mentally you can sometimes round the dividend and adjust to maintain the equality.

Example 1

Add 6 to 414.

When 420 is divided by 6, 6 ÷ 6 is added. Subtract 6 ÷ 6 to balance the amount added.

$$\begin{aligned}414 \div 6 &= (420 \div 6) - (6 \div 6)\\ &= 70 - 1\\ &= 69\end{aligned}$$

Example 2

Subtract 12 from 212.

When 200 is divided by 4, 12 ÷ 4 is subtracted. Add 12 ÷ 4 to balance the amount subtracted.

$$\begin{aligned}212 \div 4 &= (200 \div 4) + (12 \div 4)\\ &= 50 + 3\\ &= 53\end{aligned}$$

Special Quotients

To divide a whole number ending in a zero or several zeros by 10, 100, or 1000, you can remove 1, 2, or 3 zeros from the number.

Example 1

$$\begin{aligned} 2300 \div 10 &= 2300 - 1 \text{ zero} \\ &= 230 \end{aligned}$$

Example 2

$$\begin{aligned} 900 \div 100 &= 900 - 2 \text{ zeros} \\ &= 9 \end{aligned}$$

To divide dividends ending in zero or several zeros by a digit, divide the front digits and add the appropriate number of zeros.

Example 1

$$\begin{aligned} 2500 \div 5 &= (25 \div 6) + 2 \text{ zeros} \\ &= 5 + 2 \text{ zeros} \\ &= 500 \end{aligned}$$

Example 2

$$\begin{aligned} 36\,000 \div 6 &= (36 \div 6) + 3 \text{ zeros} \\ &= 6 + 3 \text{ zeros} \\ &= 6\,000 \end{aligned}$$

When both the dividend and divisor end in zero or several zeros, use these steps.

Step 1. Divide the front-end digits.

Step 2. Cancel equal numbers of zeros in the dividend and the divisor if possible.

Step 3. Add the remaining zeros to the quotient.

Example 1

$$\begin{aligned} 5400 \div 90 &= 540 \div 9 \leftarrow 540\cancel{0} \div 9\cancel{0} \\ &= (54 \div 9) + 1 \text{ zero} \\ &= 6 + 1 \text{ zero} \\ &= 60 \end{aligned}$$

Example 2

$$\begin{aligned} 63\,000 \div 70 &= 6300 \div 7 \leftarrow 63\,00\cancel{0} \div 7\cancel{0} \\ &= (63 \div 7) + 2 \text{ zeros} \\ &= 9 + 2 \text{ zeros} \\ &= 900 \end{aligned}$$

Practice Activities

Space for Your Work

1. Write the division in 3 different ways.

   a. 105 divided by 7

   b. 3028 divided by 4

2. Use = or ≠ to make a true statement for each of the following.

   a. $20 \div 1 \bigcirc 20$

   b. $8 \div 0 \bigcirc 0$

   c. $20 \div 4 \bigcirc 4 \div 20$

   d. $(12 \div 6) \div 3 \bigcirc (12 \div 3) + (6 \div 3)$

   e. $(48 \div 12) \div 2 \bigcirc 48 \div (12 \div 2)$

Space for Your Work

3. Divide mentally.

   a. $650 \div 10$

   b. $3\,200 \div 40$

   c. $20\,000 \div 500$

   d. $\dfrac{48\,000}{6\,000}$

   e. $400 \div 200$

4. Divide mentally by the plus-minus method.

   a. $3\overline{)207}$

   b. $96 \div 2$

   c. $\dfrac{396}{4}$

Space for Your Work

5. Find the quotient mentally by the left-right method.

   a. $369 \div 9$

   b. $\dfrac{425}{5}$

   c. $8\overline{)656}$

6. Divide mentally, using a method of your choice.

   a. $\dfrac{1836}{6}$

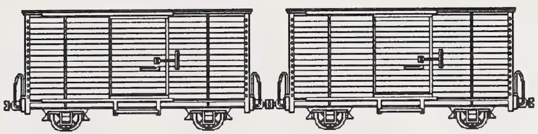
   b. $5\overline{)2980}$

   c. $558 \div 9$

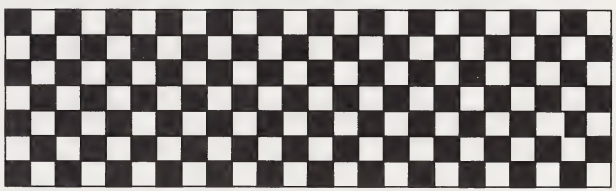
Space for Your Work

7. Solve mentally.

a. If there are 12 000 tonnes of grain in 40 boxcars, how many tonnes are in each boxcar?



b. If 168 tiles will cover the floor and there are 7 tiles in a column, how many columns of tiles are there?



See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Find the missing quotient, divisor, or dividend.

   a. $31\,000 \div \boxed{} = 310$

   b. $\dfrac{\boxed{}}{1} = 4$

   c. $5\,\overline{)\,6000}$ with quotient $\boxed{}$

   d. $\boxed{} \div 400 = 30$

   e. $\boxed{}\,\overline{)\,810\,000}$ with quotient 900

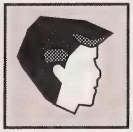
2. Divide mentally, using the methods you learned in this lesson.

   a. $84 \div 7$

   b. $4\,\overline{)\,196}$

   c. $486 \div 6$

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Division can be thought of as successive subtraction.

For example, if I have 15 chocolates, I can give 3 chocolates each to how many people?

$15 - 3 = 12$ There are 12 left after giving 3 to the first person.

$12 - 3 = 9$ There are 9 left after giving 3 to the second person.

$9 - 3 = 6$ There are 6 left after giving 3 to the third person.

$6 - 3 = 3$ There are 3 left after giving 3 to the fourth person.

$3 - 3 = 0$ There are none left after giving 3 to the fifth person.

$$15 - \underbrace{3 - 3 - 3 - 3 - 3}_{\text{5 subtractions}} = 0 \quad \text{so} \quad 15 \div 3 = 5$$

Now consider this problem.

I have 15 chocolates. To how many people can I give 0 chocolates each?

$15 - 0 = 15$ There are 15 left after giving 0 to the first person.

$15 - 0 = 15$ There are 15 left after giving 0 to the second person.

$15 - 0 = 15$ There are 15 left after giving 0 to the third person.

⋮

$15 - 0 = 15$ There are 15 left after giving 0 to the hundredth person.

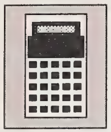
⋮

No matter how many people there are, the chocolates would not be exhausted. So you can see that division by zero does not lead to a unique answer. It is for this reason that division by zero is **undefined**.

Concluding Activities

Space for Your Work

Use a calculator for this activity.



1. Many people make the error of thinking that dividing by zero should give zero for an answer. Here is an investigation to help you think about that idea. With a calculator, do the following series of divisions in the order shown, and record your answers.

   a. $100 \div 100$

   b. $100 \div 50$

   c. $100 \div 25$

   d. $100 \div 20$

   e. $100 \div 10$

   f. $100 \div 5$

   g. $100 \div 4$

   h. $100 \div 2$

   i. $100 \div 1 =$

Space for Your Work

2. Notice that the divisors in Question 1 are a set of numbers which become smaller and smaller. That is, they approach zero.

   What pattern or trend can you see in the answers? What does this trend suggest about division by zero and the possibility of getting zero as an answer?

3. Enter any whole number, divide it by 0, and press $\boxed{=}$. What appears on the display? Why?

See your learning facilitator to check your answers and to receive further instructions.

ORDER OF OPERATIONS

What Lies Ahead

In this section you will learn this skill.

- using the correct order of operations in calculations

In this section you will use these words.

- parentheses (brackets)
- evaluate

Working Together

Have you ever answered a skill-testing question in a contest? These questions usually involve order of operations.

Example

> "The contest winner must first correctly answer this skill-testing question:
>
> Find the value of $54 \div 6 + 2 \times 4 - 4$."

$$
\begin{aligned}
& 54 \div 6 + 2 \times 4 - 4 \\
= & \quad 9 \quad + \quad 8 \quad - 4 \\
= & 17 - 4 \\
= & 13
\end{aligned}
$$

The answer is 13.

Did you notice the operations were not done simply from left to right? Special rules for the order or operations were followed.

The steps for the order of operations are as follows:

Step 1. Perform operations in **brackets**.

Step 2. Evaluate **exponents**.

Step 3. Do **division/multiplication** from left to right.

Step 4. Do **addition/subtraction** from left to right.

The acronym "BEDMAS" will help you remember this order. Use the first letters from these words to get BEDMAS.

B rackets
E xponents
D ivision
M ultiplication
A ddition
S ubtraction

Note

- An acronym is a word formed from the first letters or syllables of other words.
- These brackets, ( ), are called **parentheses**.

Example 1

Find the value of $(4 \times 8) - 6 \div 2 \times 5$.

$$\begin{aligned} & (4 \times 8) - 6 \div 2 \times 5 \\ &= 32 - 6 \div 2 \times 5 \\ &= 32 - 3 \times 5 \\ &= 32 - 15 \\ &= 17 \end{aligned}$$

Note

- First the operation in brackets was performed.
- Then the division was done.
- Next the multiplication was done.
- Finally, the subtraction was done.

Example 2:

Find the value of $48 \div 6 + 2 \times 5 - 4$.

$$
\begin{aligned}
& 48 \div 6 + 2 \times 5 - 4 \\
&= 8 + 2 \times 5 - 4 \\
&= 8 + 10 - 4 \\
&= 18 - 4 \\
&= 14
\end{aligned}
$$

Note

- First the division was done.
- Next the multiplication was done.
- Then the addition was done.
- Finally, the subtraction was done.

Example 3:

Evaluate $\dfrac{8 + 9 \times 3}{23 - 36 \div 2}$

Evaluate means "find the value of."

$$
\begin{aligned}
& \frac{8 + 9 \times 3}{23 - 36 \div 2} \\
&= (8 + 9 \times 3) \div (23 - 36 \div 2) \\
&= (8 + 27) \div (23 - 18) \\
&= 35 \div 5 \\
&= 7
\end{aligned}
$$

Note

- First the division was rewritten with brackets.
- Then the multiplication and division within the brackets were done.
- Next the addition and subtraction within the brackets were completed.
- Finally, the division was performed.

Practice Activities

Space for Your Work

1. Evaluate each of the following.

   a. $2 \times 24 \div 6 - 6 + 6$

   b. $2 + 8 \div 2 - 5$

   c. $42 - (9 \times 2) \div 3$

   d. $(9 \times 2 \times 2) \div 12$

   e. $12 + 5 \times 2 - 34 \div 2$

Space for Your Work

2. Use $<$, $>$, or $=$ to make each of the following a true statement.

   a. $(8 + 3) \times 5 \bigcirc 8 + 3 \times 5$

   b. $7 + 1 \times 4 \bigcirc 5 \times 2 + 6$

   c. $4 \times 9 \div 3 \bigcirc 8 + 2 \times 2$

   d. $24 - 12 \div 6 \bigcirc 2 + 6 \times 0$

   e. $3 \times 3 \div 3 \bigcirc 3 \div 3 \times 3$

Space for Your Work

3. Evaluate each of the following.

a. $\dfrac{8 + 3 - 1}{2 \times 5 - 8}$

b. $\dfrac{32 - 8}{3 \times 3 \div 3 + 3}$

c. $\dfrac{(2 \times 3) \times 4}{10 - 2 \times 3}$

d. $\dfrac{4 + 2 \times 2 + 4}{(4 \times 6) \div 12}$

e. $\dfrac{20 - 3 \times 2}{(15 + 7) \div 11}$

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

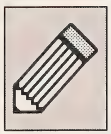
Space for Your Work

Computer Alternative

1. Do Lessons 1 and 2 of the Pre-Algebra disk from the package *Computer Drill and Practice: Mathematics*, Level D. Read the instructions in the folder with the disk before using the program. Remember if you need help or have an error hold down the SHIFT key and press the ? key.

   Note: • means multiply.

Print Alternative



2. $9 - 5 + 4 \times 9 - 26 \div 13$.

   a. Which operation is performed first?

   b. Which operation is performed next?

   c. Find the value of the expression.

3. Evaluate $16 - (8 \times 4) \div 32$.

   a. Which operation is performed first?

   b. Which operation is performed next?

   c. Find the value of the expression.

Space for Your Work

4. Find the value.

   a. $8 \times 7 + 5$

   b. $9 \times 5 \div 3$

   c. $32 + 8 \div 4$

   d. $56 \div 7 \div 8$

   e. $(175 + 12) \times 10$

   f. $29 - 16 \div 4 + 7$

   g. $5 \times 6 \div (13 - 10)$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

You can evaluate a series of operations on a calculator. However, not all calculators work the same way. Compare how you compute a series of operations using paper and pencil, a regular calculator, and a scientific calculator.

Example: $7 + 6 \times 3$

- using with paper and pencil

$$\begin{aligned} & 7 + 6 \times 3 \\ &= 7 + 18 \\ &= 25 \end{aligned}$$

- using a regular calculator

Key Press	Display
6 × 3 =	18
+ 7 =	25

- using with a scientific calculator

Key Press	Display
7 + 6 × 3 =	25

Note

- With the paper-and-pencil method and with the regular calculator, rules of order of operation were used. That is, multiplication was done first.

  The scientific calculator allows you to press the keys in the order that the series of operations is written.

Be careful of number sentences which have brackets. Regardless of the type of calculator you have, more than one step may be needed.

Example: $7 \times (5 - 3)$

- with paper and pencil

$$7 \times 5 - 3)$$
$$= 7 \times 2$$
$$= 14$$

Note

- Operations in brackets are performed first.

- with a calculator

Key Press	Display
5 – 3 =	2
× 7 =	14

Note

- Operations in brackets are performed first.

A calculator's memory can help you evaluate a series of operations. The memory on a calculator allows you to store numbers in the calculator's memory while you work with other members. The memory keys may differ somewhat from one calculator to another, but these four keys are frequently used:

M+	adds the number in the display to the number in the calculator's memory.
M–	subtracts the number in the display from the number in the calculator's memory.
MR	recalls the number in the memory to the display.
MC	clears the number in the memory.

Example 1: $7 + 6 \times 3$

Key Press	Display
7 M+	M 7
6 × 3 = M+	M 18
MR	M 25

Note

- 7 is placed in the memory.
- Next, the product of 6 and 3 is added to the number in the memory.
- Finally, the number in the memory is recalled to the display.

Example 2: $4 + 5 - (3 + 2)$

Key Press	Display
4 + 5 = M+	M 9
3 + 2 = M–	M 5
MR	M 4

Note

- The sum of 4 and 5 is placed in the memory.
- Then the sum of 3 and 2 is subtracted from the number in the memory.
- Finally, the number in the memory is recalled.

Concluding Activities

Space for Your Work

1. Evaluate the following on paper first. Then use your calculator to get the same result. Remember to clear the calculator display before computing. To do this press C.

   a. $12 + 5 \times 2$

   b. $18 + 6 \div 3$

   c. $19 - 6 \times 2 + 3$

   d. $20 - 8 \div 4 - 5$

Space for Your Work

2. Evaluate the following on paper first. Then use your calculator to get the same result. Remember to clear the calculator display before computing. To do this press $\boxed{\text{C}}$.

   a. $(5 + 3) \times (9 - 2)$

   b. $(3 + 5 - 2) \div (7 - 5)$

   c. $156 \div (29 - 17)$

   d. $15 - (13 - 2)$

Space for Your Work

3. Evaluate the following on paper first. Then use your calculator to get the same answer. Remember to clear the calculator's memory and the calculator display before computing. Press $\boxed{MC}$ to clear the memory.

   a. $9 + 3 \times 15 - 11$

   b. $(8 + 9) - (7 + 2)$

   c. $(12 - 6) \times (10 - 7)$

   d. $7 \times (5 - 3)$

4. Use parentheses to make a true statement.

   a. $5 \times 2 - 1 = 5$

   b. $28 - 12 \div 4 = 4$

   c. $7 - 5 \times 12 = 24$

   d. $38 - 13 + 17 - 12 = 20$

   e. $54 \div 12 + 6 \times 6 = 18$

   f. $8 \div 4 + 2 \div 2 + 9 \div 3 + 3 \div 3 - 4 = 0$

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this summary you will review the skills you learned in Part Two.

- computing mentally exact whole numbers, sums, differences, products, and quotients
- estimating whole number sums, differences, products, and quotients
- using the rules for order of operation

Working Together

At this point, it is a good idea to review the skills you have learned in Part Two.

Turn to Section 14 and review the Pretest. Correct any errors you may have made. You may be pleasantly surprised to discover how much you have learned.

PART THREE

WESTFILE, INC.

Parts One and Two dealt with operations on whole numbers. Part Three deals with multiples, factors, and powers of whole numbers.

Part Three also deals with integers. Integers can be used to describe many situations.

- How high is the mountain?
- How low is the valley?
- How hot is it inside the house?
- How cold is it outside the house?

GETTING SET

What Lies Ahead

This section will test these skills.

- identifying multiples of whole numbers
- finding multiples, common multiples, and LCM
- finding factors, common factors, and GCF
- classifying numbers as prime or composite
- writing a number as a product of prime factors
- deciding if a number is divisible by 2, 3, 5, 6, 9, or 10
- recognizing powers
- writing the value of a power
- writing numbers in expanded form using powers
- recognizing an integer
- comparing and ordering an integer
- adding integers using counters and number lines

Working Together

The pretest will help you and your learning facilitator identify your strengths and weaknesses.

Pretest

Space for Your Work

1. Give the first five multiples of the number.

   a. 2

   b. 3

   c. 4

2. Is the second number a multiple of the first?

   a. 3, 24

   b. 7, 74

   c. 35, 5

3. Give all the factors of the numbers.

   a. 14

   b. 26

   c. 42

Space for Your Work

4. Which are prime numbers?

   a. 5

   b. 14

   c. 23

   d. 42

5. Find two prime numbers whose sum is a composite number.

6. Find the prime factorization of each number.

   a. 34

   b. 54

   c. 50

Space for Your Work

7. Is 17 245 divisible by each of the numbers? Do not divide or use a calculator.

   a. 2

   b. 3

   c. 5

   d. 9

   e. 10

8. Write each of these numbers as a power of ten.

   a. 100

   b. 1000

   c. 10 000

9. Name the base and the exponent in each power given.

   a. 10^1

   b. 25^{100}

Space for Your Work

10. Express each number given as a power.

    a. 25

    b. 32

    c. 81

11. Find the value of each power given.

    a. 7^2

    b. 10^6

    c. 4^3

12. Represent each of the following by a positive or negative number.

    a. 10°C below freezing

    b. 5 floors above ground level

    c. 2 under par

    d. 80 m below sea level

    e. $92 overdrawn

Space for Your Work

13. Use > or < to make a true statement for each of these.

a. $-6 \bigcirc +1$

b. $+15 \bigcirc -2$

c. $-8 \bigcirc -5$

d. $+4 \bigcirc +3$

14. Write the opposite of each of the following.

a. $+3$

b. -5

c. 0

Space for Your Work

15. Complete the following number sentences. Use counters or number lines to add.

a. $(+8) + (+2) = \square$

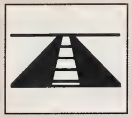
b. $(-3) + (-5) = \square$

c. $(+2) + (-7) = \square$

d. $(-4) + (+9) = \square$

See your learning facilitator to check your answers and to receive further instructions.

MULTIPLES



What Lies Ahead

In this section you will learn these skills.

- identifying multiples of whole numbers
- finding multiples of whole numbers

In this section you will use these words.

- multiple
- common multiple

Working Together

This section deals with multiples.

The word **multiple** should remind you of *multiply* and *multiplication*. The connection is a very close one, as you will see.

Using a Calculator to Find Multiples

Example 1: Find the multiples of 9 using your calculator.

Solution 1: Using Addition

Key Press	Display
9 + 9 =	18
+ 9 =	27
+ 9 =	36

Solution 2: Using Automatic Constant

Key Press	Display
9 + 9 =	18
=	27
=	36

Note

- On some calculators you need to press = twice.
- Not all calculators have the automatic feature for addition. Experiment with your calculator.

Solution 3: Using Multiplication

Key Press	Display
1 × 9 =	9
2 × 9 =	18
3 × 9 =	27

Note: It may be necessary to press C each time.

Solution 4: Using Automatic Constant

Key Press	Display
9 × 1 =	9
2 =	18
3 =	27

Note: Not all calculators have the automatic feature for multiplication. Experiment with your calculator.

Checking for Multiples

Example 2: Is 117 a multiple of 9?

Solution 1: Listing the multiples.

List the multiples using your calculator and one of the methods described in Example 1.

9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117

Yes, 117 is a multiple of 9.

Solution 2: Using Division

- using paper and pencil

$$\begin{array}{r} 13 \\ 9\overline{)117} \\ 9\downarrow \\ \hline 27 \\ 27 \\ \hline 0 \end{array}$$

- using the calculator

Key Press	Display
1 1 7 ÷ 9 =	13

The division shows no remainder so 117 is a multiple of 9.

Example 3: Is 121 a multiple of 6?

Solution 1: Listing the multiples

List the multiples using your calculator and one of the methods described in Example 1.

6, 12, 18, 24, 36, 42, 48, 54, 60, 66, 72,
78, 84, 90, 96, 102, 108, 114, 120, 126

No, 121 is not a multiple of 6.

Solution 2: Using Division

- using paper and pencil

$$\begin{array}{r} 20 \\ 6\overline{)121} \\ 12 \\ \hline 1 \\ 0 \\ \hline 1 \end{array}$$

- using the calculator

Key Press	Display
1 2 1 ÷ 9 =	20.166666

The division shows a remainder so 121 is not a multiple of 6.

Practice Activities

Space for Your Work

1. Give the first five multiples of each number given.

   a. 3

   b. 5

   c. 6

   d. 7

2. Is the first number a multiple of the second number?

   a. 94, 7

   b. 86, 4

   c. 98, 12

   d. 56, 14

   e. 35, 7

   f. 49, 6

Space for Your Work

3. a. Is 6 a multiple of 3?

   b. Is 3 a multiple of 6?

   c. Is 297 a multiple of 3?

   d. Is 297 a multiple of 6?

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Give the first six multiples of each of the following numbers.

   a. 2

   b. 4

   c. 12

2. Is the first number a multiple of the second number?

   a. 86, 2

   b. 26, 4

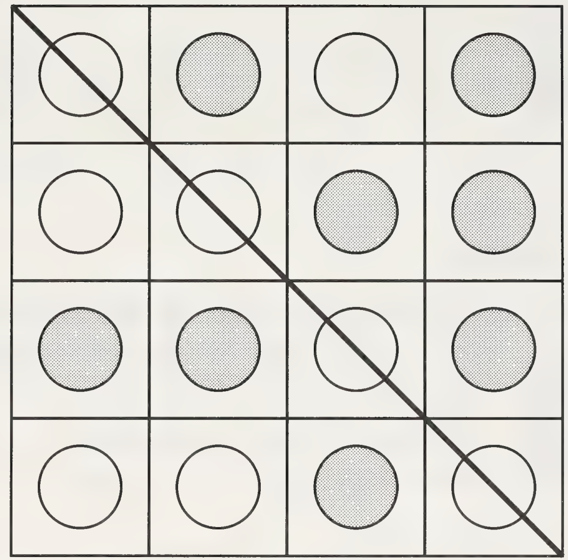
   c. 23, 3

   d. 75, 5

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Multiple Tic-Tac-Toe is a game for two players.



How to Play Multiple Tic-Tac-Toe

Step 1. Players roll the die to see who goes first. Player who rolls the higher number goes first.

Step 2. In turn, each player throws the die and covers a multiple of that number on the game board.

If ⚅ is rolled, the player can cover 6, 12, 24, or 30. (Cover one only.)

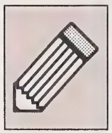
Step 3. The game continues until one player is able to place four disks in a row horizontally, vertically, or diagonally.

Concluding Activities

Computer Alternative

1. Play the game "Multiples" on the *Number Munchers* disk.

Print Alternative



2. Play the game "Multiple Tic-Tac-Toe" with another person. You will need one die and several 2-coloured disks. Use the game board provided in the appendix of the module booklet.

See your learning facilitator to check your answers and to receive further instructions.

FACTORS

What Lies Ahead

In this section you will learn these skills.

- finding the factors of a number
- finding the greatest common factor of two or more numbers

In this section you will use these words.

- factors
- common factors
- greatest common factor

Working Together

This section deals with factors.

Earlier in the module you reviewed the operation of multiplication. You learned that factors can be multiplied to give you a product.

Example: $\underbrace{2 \times 5}_{\text{factors}} = \underbrace{10}_{\text{product}}$

In this section you will discover how to find the factors of a number.

Learning Aids Activities

Turn to Exercise F in the *Learning Aids Booklet* and do the activities.

Working Together

Factors are used in everyday life.

Example

Ming has a butterfly collection. He wants to arrange the 12 butterflies in rows with the same number of butterflies in each row. What are the different possible combinations?

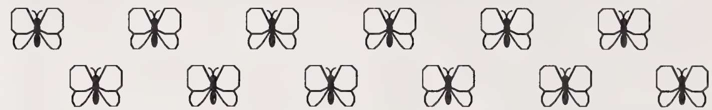
Solution: Ming can use factors to group the butterflies. He can make an organized list to find all the different factors.

12	
1	12
2	6
3	4
4	3
5	← 5 is not a factor.
6	2
7	← 7 is not a factor.
8	← 8 is not a factor.
9	← 9 is not a factor.
10	← 10 is not a factor.
11	← 11 is not a factor.
12	1

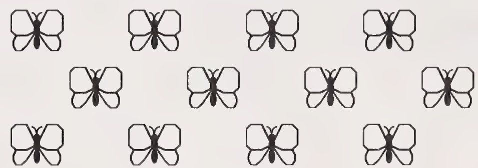
The butterflies can be arranged as these illustrations show.

Since $1 \times 12 = 12$, he gets 1 row of 12.

Since $2 \times 6 = 12$, he gets 2 rows of 6.



Since $3 \times 4 = 12$, he gets 3 rows of 4.



There are three other possible arrangements.

- 12 rows of 1 each
- 4 rows of 3 each
- 6 rows of 2 each

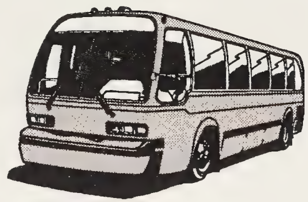
Practice Activities

Space for Your Work

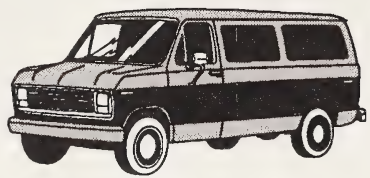
Use whole numbers to answer Question 1.

1. There are 168 people going skiing.

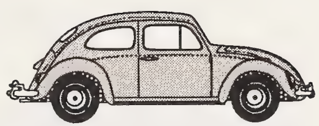
   a. A bus can take 42 people. How many buses would be needed to provide transportation for all the people going skiing?



   b. A van can take 12 people. How many vans would be needed to accommodate all the people going skiing?



   c. A car can take 4 people. How many cars would be needed to take all the skiers skiing?



Space for Your Work

2. Find all the factors of the numbers.

   a. 30

   b. 32

   c. 59

   d. 70

Space for Your Work

3. Does each of the following numbers have 8 as a factor?

   a. 30

   b. 40

   c. 87

   d. 144

4. Complete with the word "factor" or "multiple."

   a. 3 is a ______________________ of 12 and

      12 is a ______________________ of 3.

   b. 12 is a ______________________ of 4 and

      4 is a ______________________ of 12.

5. What number is a factor of every number?

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Computer Alternative

1. Play the game "Factors" on the *Number Munchers* disk.

Print Alternative

2. Copy and supply the missing factors.

   a. $1 \times \square = 20$

   b. $2 \times \square = 20$

   c. $4 \times \square = 20$

   d. $5 \times \square = 20$

   e. Give all the factors of 20.

Space for Your Work

3. a. What number do you try first when finding all the factors of 28?

   b. Do you get a factor?

   c. What number do you try next?

   d. Do you get a factor?

   e. Continue until you have all the factors of 28.

4. a. What are the factors of 36?

   b. What are the factors of 45?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Tax Collector is a game for two players.

How to Play Tax Collector

Step 1. You choose the number of coins you want for the first round. (Pick a number from 6 to 25.) Lay the coins on the table in front of you. Begin with 1 and arrange the coins in order until you have the number you chose.

Step 2. You pick a coin that has a factor showing. You give the tax collector all of the coin's other factors.

Step 3. You pick another coin that has a factor showing. You pay the tax collector all of the coin's other factors.

Step 4. You continue picking coins with factors showing and paying the tax collector. When you cannot pick any more, the tax collector gets the remaining coins.

Step 5. Total your score and the tax collector's score. Did you beat the tax collector?

See the example on the next page.

Example

Step 1. You choose to play with 8 coins for the first round.

1 2 3 4 5 6 7 8

Step 2. You pick 6 and keep it for yourself. You pay the tax collector 3, 2, 1 since these are the other factors of 6. This leaves these coins to play with.

4 5 7 8

Step 3. You cannot pick 4, 5, 7 as they do not have any factors showing. You pick 8 and keep it for yourself. You pay the tax collector 4 since 4 is a factor of 8. This leaves these coins to play with.

5 7

Step 4. You cannot pick any more coins with factors showing since 5 is not a factor of 7 and 7 is not a factor of 5. The tax collector gets 5, 7.

Step 5. To find your score, add the numbers you picked to get $6 + 8 = 14$. To find the tax collector's score, add the rest of the numbers to get $1 + 2 + 3 + 4 + 5 + 7 = 22$. This means that the tax collector won the game.

Concluding Activities

Computer Alternative

1. Play the game "Tax Collector" on the *Conquering Whole Numbers* disk.

Print Alternative

2. Play the game "Tax Collector" with a partner.

   You will need the coins which are found in the appendix of the module booklet.

See your learning facilitator to check your answers and to receive further instructions.

PRIME AND COMPOSITE NUMBERS

What Lies Ahead

In this section you will learn this skill.

- classifying numbers as prime or composite

In this section you will use these words.

- factor
- composite number
- prime number

Working Together

In this section you will learn about prime and composite numbers.

Learning Aids Activities

To prepare yourself for this section, turn to Exercise G in the *Learning Aids Booklet* and complete the activities.

A number with exactly two different factors, 1 and itself, is a **prime number**.

$$5 = 5 \times 1$$

5 is a prime number since it has factors of 1 and 5.

A number with more than two different factors is a **composite number**.

$$8 = 8 \times 1$$

$$8 = 4 \times 2$$

8 is a composite number since it has factors of 1, 2, 4, and 8.

Working Together

A scholar named Eratosthenes, an ancient Greek mathematician, invented a method for listing prime numbers in order.

Try his method which is explained here.

The Sieve of Eratosthenes can be used to find prime numbers less than a given number. For example, to find all the prime numbers less than 50:

- Circle 2 and cross out all the multiples of 2.
- Circle 3 and cross out all the multiples of 3.
- Circle 5 and cross out all the multiples of 5.
- Circle 7 and cross out all the multiples of 7.
- Circle the remaining numbers.

The Sieve of Eratosthenes

1	2	3	~~4~~	5	~~6~~
7	~~8~~	~~9~~	~~10~~	11	~~12~~
13	~~14~~	~~15~~	~~16~~	17	~~18~~
19	~~20~~	~~21~~	~~22~~	23	~~24~~
~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~
37	~~38~~	~~39~~	~~40~~	41	~~42~~
43	~~44~~	~~45~~	~~46~~	47	~~48~~
~~49~~	~~50~~				

Note

- Crossed out numbers are composite.
- Circled numbers are prime.
- The number 1 has only **one** factor, itself, so it can be called neither prime nor composite. The number 1 belongs to a class of its own.

Practice Activities

1. a. Complete the table.

Space for Your Work

Number	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Space for Your Work

b. What type of number has exactly two factors?

c. What type of number has more than two factors?

d. Make a list showing the smallest number having exactly 1 factor, exactly 2 factors, and so on up to 6 factors.

Use the Sieve of Eratosthenes to answer Questions 2-5.

2. Find a pair of prime numbers that has a difference of 1.

3. A pair of prime numbers which has a difference of 2 is called a **twin prime**.

   How many twin primes are there between 2 and 25?

Space for Your Work

4. What is the sum of the 1st, 2nd, 3rd, and 4th primes? Is this sum prime?

5. Find four pairs of prime numbers where each pair has a sum of 50.

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

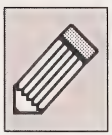
Space for Your Work

Computer Alternative

1. Do Lesson 14 of the ''Numbers and Numeration'' disk of the package *Computer Instruction and Drill: Mathematics*, Level D. Read the instructions in the folder with the disk before you use the program. Remember, if you need help or have an error, hold down the SHIFT key and press the ? key.

2. Play the game ''Primes'' on the *Number Munchers'' disk.*

Print Alternative



3. a. What is your age?

   b. Is your age a prime or a composite number? Why?

4. Is each of the following numbers prime or composite? Tell why in each case.

   a. 6

   b. 11

   c. 29

   d. 33

   e. 50

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. The numbers 11 and 101 are prime.

   a. Is 1001 prime?

   b. Is 10 001 prime?

   c. Is 100 001 prime?

2. A mathematician named Christian Goldbach theorized in 1742 that every even natural number is equal to the sum of two primes. Test his theory using each of the following numbers.

   a. 4

   b. 6

   c. 8

   d. 10

   e. 12

Space for Your Work

f. 20

g. 26

h. 38

i. 50

j. 72

Space for Your Work

3. Christian Goldbach also believed that a natural number greater than 7 (even or odd) is equal to the sum of 3 primes. Test his theory using the following numbers.

   a. 8

   b. 9

   c. 10

   d. 11

   e. 12

Space for Your Work

f. 17

g. 33

h. 52

i. 75

j. 101

See your learning facilitator to check your answers and to receive further instructions.

PRIME FACTORS

What Lies Ahead

In this section you will learn these skills.

- recognizing prime factors
- expressing a number as a product of prime factors

In this section you will use these words.

- prime factor
- prime factorization
- factor tree

Working Together

A **factor** is a number used to make a product. A **prime number** is a number which has only two factors (1 and itself).

Can you now tell what a prime factor is?

Consider the number 15.

Its factors are 1, 3, 5, and 15.

Two of these factors are prime. They are 3 and 5.

3 and 5 are prime, and they are factors of 15. So, we say that 3 and 5 are the prime factors of 15.

A **prime factorization** is an expression of a number as the product of its prime factors.

The prime factorization of 15 is $15 = 3 \times 5$.

Any number can be written as a product of prime factors. This task becomes more difficult with larger numbers. In this section you will discover various methods which can be used.

Here are two methods for finding the prime factorization of a number. These methods are making a factor tree and dividing by prime factors.

Making a Factor Tree

Here is how a factor tree is made.

- Start with any pair of factors for the number and factor them also if possible.
- Continue to factor until you get prime factors.

Example

Express 36 as the product of prime factors.

$$\begin{array}{c} 36 \\ 12 \times 3 \\ 3 \times 4 \times 3 \\ 3 \times 2 \times 2 \times 3 \end{array} \qquad \text{OR} \qquad \begin{array}{c} 36 \\ 18 \times 2 \\ 2 \times 9 \times 9 \\ 2 \times 3 \times 3 \times 2 \end{array}$$

The prime factorization of 36 = 2 × 2 × 3 × 3.

Note

Factors in the prime factorization are usually written from smallest to largest.

Dividing by Prime Factors

Here is how division by prime factors is done.

- Divide the number by the lowest prime number that is a factor of this number.
- Continue to divide the resulting quotient in the same way until the quotient is one.

Example

Express 36 as the product of prime factors.

$$\begin{array}{l} 36 \div 2 = 18 \\ 18 \div 2 = 9 \\ 9 \div 3 = 3 \\ 3 \div 3 = 1 \end{array} \qquad \text{OR} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

The prime factorization of 36 = 2 × 2 × 3 × 3.

Practice Activities

Space for Your Work

1. The factors of 14 are 1, 2, 7, 14. Which are prime factors?

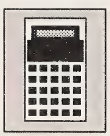
2. Write the prime factors of each number.

   a. 10

   b. 16

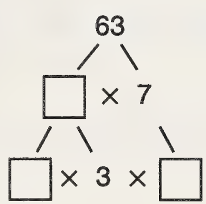
   c. 24

Use a calculator to help find the factors in Questions 3-7.

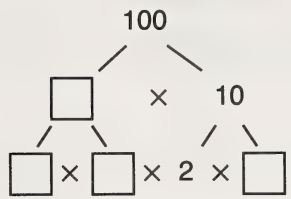


3. Complete the factor tree in each of the following.

   a.



b.



4. Make a factor tree to find the prime factorization for each of these numbers.

   a. 38

   b. 42

Space for Your Work

c. 50

d. 72

5. a. What is the least prime number? Is it a factor of 75?

   b. What is the least prime number that is a factor of 75?

   c. Divide to find all the prime factors of 75.

Space for Your Work

6. Divide by prime factors to find the prime factorization for each of these numbers.

   a. 99

   b. 100

   c. 108

   d. 110

7. Find the prime factorization for each of these numbers.

   a. 39

   b. 56

   c. 60

   d. 288

See your learning facilitator to check your answers and to receive further instructions.

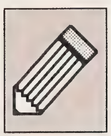
Extra Practice

Space for Your Work

Computer Alternative

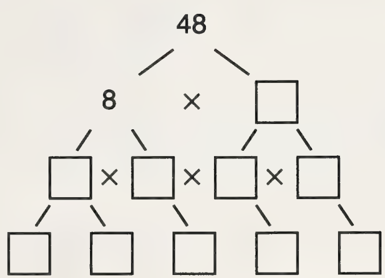
1. Do Lesson 15 of the disk "Numbers and Numeration" for the package *Computer Drill and Instruction: Mathematics,* Level D (SRA). Read the instructions in the folder with the disk. Remember, if you need help or have an error, hold down the SHIFT key and press the ? key.

Print Alternative



2. Complete.

   a.



Space for Your Work

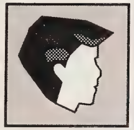
b.

$$\begin{array}{ccccc} & & 30 & & \\ & / & & \backslash & \\ & \square & \times & 5 & \\ / & \backslash & & \backslash & \\ \square & \times \ \square & \times & \square & \end{array}$$

c. $24 = \square \times \square \times \square \times \square$

d. $75 = \square \times \square \times \square$

See your learning facilitator to check your answers and to receive further instructions.



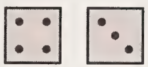
Working Together

Place Roll[1] is a game for two people.

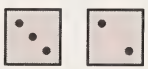
How to Play Place Roll

Step 1. Choose the starting player by rolling the dice. The player with the highest number goes first.

Step 2. The starting player rolls the dice and forms a number with the top two faces. The larger number is in the tens position and the smaller number is in the ones position.

Example:  = 43

Step 3. The player then puts one disc and only one in any box on the playing board that contains a factor of that number.

Example:  = 32

The player can cover 8, 4, 2, or 16.

Step 4. If the player rolls a double such as 33, 22, or 11, the turn is lost.

Step 5. If the player rolls a prime such as 61, 53, 41, 43, or 31, he or she can place a disk on any "prime" box.

Step 6. If a box is already occupied, a player cannot use it.

Step 7. The winner is the first player to cover four boxes in a row.

[1]National Council of Teachers of Mathematics for excerpts from *The Arithmetic Teacher*, September, 1987, Reston, Virginia.

Concluding Activities

Play the game "Place Roll".

You will need a pair of dice, several disks or bingo chips of 2 colours and a game board. The game board is found in the appendix.

See your learning facilitator to check your answers and to receive further instructions.

DIVISIBILITY

What Lies Ahead

In this section you will learn this skill.

- deciding if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10

In this section you will use this word.

- divisible

Working Together

Divisibility?

Is that what happens when you're driving down the road in a fog, and da-visibility is bad??

This section will try to clear away the fog for you!

One number is divisible by another if the dividing gives a remainder of zero.

The chart on the next page provides helpful tests to show whether one number is divisible by another.

A number is divisible by	when	Example
2	the number is even (ends in 0, 2, 4, 6, or 8).	6934 is divisible by 2 since it ends in a 4.
3	the sum of the digits is divisible by 3.	8754 is divisible by 3 since 8 + 7 + 5 + 4 = 24 and 24 is divisible by 3.
4	the last two digits are divisible by 4.	17 324 is divisible by 4 since the last two digits put together make 24 which is divisible by 4.
5	the last digit is 0 or 5.	17 365 is divisible by 5 since the last digit is 5.
6	the number is divisible by both 2 and 3.	2382 is divisible by 6 because 2382 is divisible by 2, and 2382 is divisible by 3 since 2 + 3 + 8 + 2 = 15 and 15 is divisible by 3.
8	the last three digits are divisible by 8.	159 128 is divisible by 8 since 128 ÷ 8 = 16.
9	the sum of digits is divisible by 9.	864 is divisible by 9 since 8 + 6 + 4 = 18 and 18 is divisible by 9.
10	the last digit is 0.	320 is divisible by 10 since the last digit is 0.

These tests for divisibility can be helpful in solving problems. Consider the examples on the next page.

Example 1

Holly is cutting pieces of paper 6 cm long from a roll of paper 750 cm long to make paper flowers. Can she cut the roll into 6-cm pieces without any waste?

Solution

Is 750 divisible by 6?

750 is even, so it is divisible by 2.

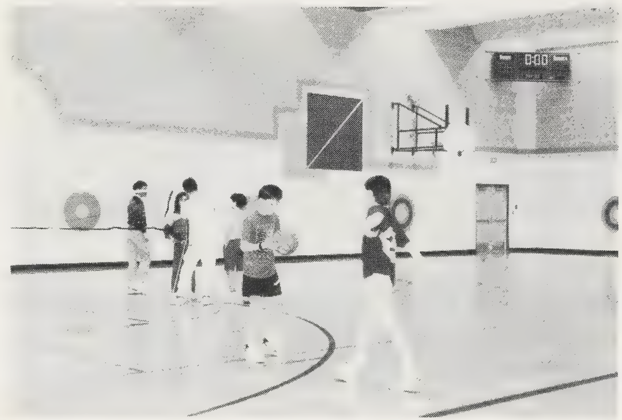
$7 + 5 + 0 = 12$ and 12 is divisible by 3, so 750 is divisible by 3.

750 is divisible by 2 and 3 so it is divisible by 6.

The roll can be cut into 6-cm pieces without any waste.

Example 2

There are 675 students in Cobequid Junior High School. The principal wishes to group the students into teams for intramural sports. Each team must have the same number of students. Can the teams have exactly 9 students each?



Solution

Is 675 divisible by 9?

$6 + 7 + 5 = 18$ and 18 is divisible by 9 so 675 is divisible by 9.

Each team can have exactly 9 students each.

Practice Activities

Space for Your Work

1. Which of these numbers is a multiple of 2?

   a. 7434

   b. 8259

   c. 15 784

   d. 156 432

   e. 24 996

   f. 313 277

2. Does each of these numbers have 3 as a factor?

   a. 6948

   b. 38 117

   c. 938 157

   d. 90 928

   e. 85 524

   f. 283 146

Space for Your Work

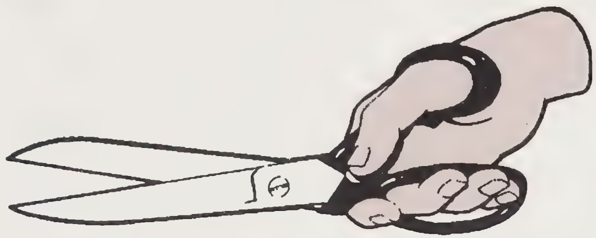
3. Does each of these numbers have 5 as a factor?
   a. 19 420
   b. 56 006
   c. 84 757
   d. 92 525
   e. 52 556
   f. 199 340

4. Which numbers in question 2 are divisible by 6?

5. Is each of these numbers a multiple of 10?
   a. 888
   b. 2835
   c. 89 030
   d. 795 485
   e. 277 102
   f. 111 220

Space for Your Work

6. Nadine has a piece of material 35 m long to make batik prints. Each batik print requires 6 m of material. Can Nadine use all of the material? If not, how much is wasted?



See your learning facilitator to check your answers and to receive further instructions.

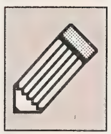
Extra Practice

Space for Your Work

Computer Alternative

1. Do Lesson 13 of the disk "Numbers and Numeration" from the package *Computer Drill and Instruction: Mathematics,* Level D. Read the instructions in the folder with the disk. Remember, if you need help or have an error, hold down the SHIFT key and press the ? key.

Print Alternative



2. a. Add the digits in 521 361.

   b. Divide the sum by 3.

   c. What is the remainder?

   d. Is 521 361 divisible by 3?

   e. Is 521 361 divisible by 6? Why?

Space for Your Work

3. In each of these cases is the remainder 0? Use the divisibility rules.

   a. $762 \div 10$

   b. $2429 \div 3$

   c. $798 \div 6$

   d. $3336 \div 9$

   e. $841 \div 2$

   f. $6051 \div 5$

4. Name three 4-digit numbers that are divisible by 9.

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

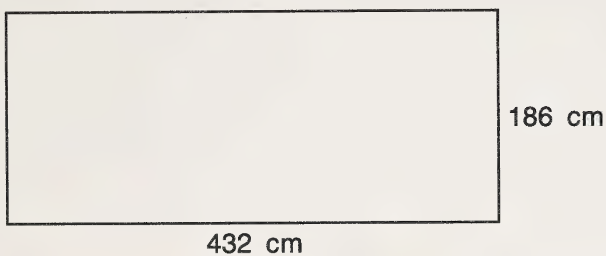
Space for Your Work

1. a. If a number is divisible by 2, 5, and 10, what must the last digit be?

   b. What do the divisibility tests for 3 and 9 have in common?

2. A number is divisible by 6 if it is divisible by 2 and 3. Try to discover a divisibility test for 15.

3. Rosita has a pane of glass 186 cm by 432 cm. She wants to cut this pane of glass into rectangular pieces measuring 3 cm by 4 cm. Can she use all of the glass?



See your learning facilitator to check your answers and to receive further instructions.

POWERS

What Lies Ahead

In this section you will learn these skills.

- recognizing numbers written as powers
- writing the value of a power
- writing standard numbers as powers

In this section you will use these words.

- power
- base
- exponent

Working Together

This section will give you a feeling of power!

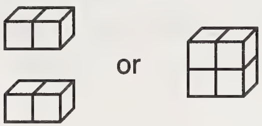
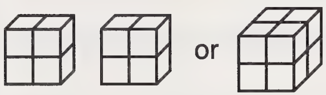
Learning Aids Activities

To prepare yourself for this section, turn to Exercise H in the *Learning Aids Booklet* and complete the activities. You will need the base 10 blocks.

In the Learning Aids activities you worked with base 10 blocks to help you understand powers.

Working Together

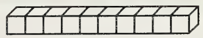
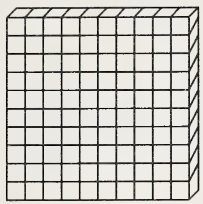
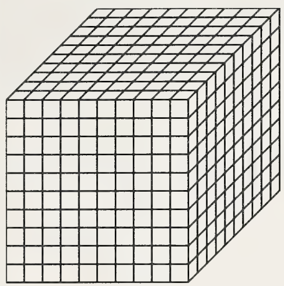
Powers of 2

Power	Concrete Model	Meaning
2^1		$2^1 = 1 \times 2$ (1 group of 2) $= 2$
2^2		$2^2 = 2 \times 2^1$ (2 groups of 2^1) $= 2 \times 2$ $= 4$
2^3		$2^3 = 2^2 \times 2^2$ (2 groups of 2^2) $= 2 \times 2 \times 2$ $= 8$

Note

- Do not confuse 2^3 and 2×3.
- 2×3 means **2 groups of 3, 3 groups of 2** or 6.
- 2^3 means **2 groups of 2^2** or $2 \times 2 \times 2 = 8$.
- 2^1 is read as 2 exponent 1, or the first power of 2.
- 2^2 is read as 2 exponent 2, or the second power of 2 or 2 squared.
- 2^3 is read as 2 exponent 3, or the third power of 2, or 2 cubed.

Powers of 10

Power	Concrete Model	Meaning
10^1		$10^1 = 1 \times 10^1$ (1 group of 10) $= 10$
10^2		$10^2 = 10 \times 10^1$ (10 groups of 10^1) $= 10 \times 10$ $= 100$
10^3		$10^3 = 10 \times 10^2$ (10 groups of 10^2) $= 10 \times 10 \times 10$ $= 1000$

Note

- 10^1 is read as 10 exponent 1, or the first power of 10.
- 10^2 is read as 10 exponent 2, or 10 squared.
- 10^3 is read as 10 exponent 3, or 10 cubed.
- 10^4 is read as 10 exponent 4, or the fourth power of 10.
- 10^5 is read as 10 exponent 5, or the fifth power of 10.

Writing Numbers as Powers

Example: Write 64 as a power.

Solution

64 can be divided by 2, 4 or 8.

$64 = 8 \times 8 = 8^2$

or $64 = 4 \times 4 \times 4 = 4^3$

or $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Example 2: Write 729 as a power.

Solution

729 can be divided by 9 or 3 (the sum of its digits equals 18).

$729 = 9 \times 9 \times 9 = 9^3$

or $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Note

Use divisibility tests to help you find a base.

Writing Numbers in Expanded Form

Powers of ten can be used to write numbers in expanded form.

Example 1: Write 5870 in expanded form.

Solution

$$\begin{aligned} 5870 &= (5 \times 1000) + (8 \times 100) = (7 \times 10) \\ &= (5 \times 10^3) + (8 \times 10^2) + (7 \times 10^1) \end{aligned}$$

Example 2: Write 72 653 in expanded form.

Solution

$$72\,653 = (7 \times 10^4) + (2 \times 10^3) + (6 \times 10^2) + (5 \times 10) + (3 \times 1)$$

Practice Activities

Space for Your Work

1. Complete the table.

Power	Base	Exponent	Meaning	Standard Form
			7×7	
3^4				
	4	3		
			6×6	
10^5				
	8			512
5^3				
	2			64

Space for Your Work

2. Express in standard form.

   a. nine squared
   b. two cubed
   c. sixth power of 10
   d. seven to the fourth power
   e. $(5 \times 10^3) + (3 \times 10^2) + (2 \times 10^1)$
   f. $(7 \times 10^3) + (8 \times 10^1) + (4 \times 1)$

3. Express as a power.

   a. 36
   b. 121
   c. 625
   d. 100

4. Write in expanded form using powers.

   a. 289 376

   b. 1 574 128

   c. 90 576

See your learning facilitator to check your answers and to receive further instructions.

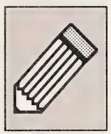
Extra Practice

Space for Your Work

Computer Alternative

1. Do Lessons 9 and 10 of the disk "Numbers and Numeration" from the package *Computer Drill and Instruction:* Mathematics, Level D. Read the instructions in the folder with the disk before using the program. If you need help or have an error hold down the SHIFT key and press the ? key.

Print Alternative



2. Which number is the exponent? Which number is the base?

   a. 2^4

   b. 9^3

   c. 5^2

   d. 3^5

   e. 10^4

   f. 6^2

Space for Your Work

3. Express the number as a power.

   a. $2 \times 2 \times 2 \times 2$

   b. $6 \times 6 \times 6$

   c. $3 \times 3 \times 3$

   d. 10×10

   e. $7 \times 7 \times 7 \times 7 \times 7$

4. Write the number in standard form.

   a. 5^2

   b. 5^3

   c. 5^4

   d. the sixth power of 5

5. Fill in the blanks. One has been done as an example.

a. $27 = \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

b. $64 = 4 \times 4 \times 4 = \_\_\_\_\_\_\_\_$

c. $1000 = \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

d. $4096 = 8 \times 8 \times 8 \times 8 = \_\_\_\_\_\_\_\_$

6. Expanded form uses powers of 10. Write the standard form.

a. $(8 \times 10^4) + (9 \times 10^3) + (3 \times 10^1)$

b. $(4 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (7 \times 1)$

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

5. a. $27 = 3 \times 3 \times 3 = 3^3$

Working Together

Calculators can be used to find powers. There are several ways to find powers. Different calculators work in different ways.

Example 1: 8^2

- using paper and pencil

$$8 \times 8 = 64$$

- using the automatic constant on a calculator

Key Press	Display
C	0
8 ×	8
=	64

- using the x^2 key on a calculator

Key Press	Display
C	0
8	8
x^2 =	64

Example 2: 8^5

- using paper and pencil

$$8 \times 8 \times 8 \times 8 \times 8 = 32\,768$$

- using the automatic constant on a calculator

Key Press	Display
C	0
8 ×	8
=	64
=	512
=	4096
=	32 768

- using the y^x key on a calculator

Key Press	Display
C	0
8	8
y^x 5 =	32 768

Concluding Activities

Space for Your work

1. Evaluate the following on paper. Then use your calculator to get the same answer.

   a. 7^2

   b. 158^2

   c. 5^3

   d. 25^4

   e. 4^6

2. A number that can be written with an exponent of 2 is called a square. For example, 36 is a square because it can be expressed as 6^2. List 5 other squares.

Space for Your Work

3. There are many different expressions for 81. Here are a few.

3×27	$243 \div 3$
9×9	$405 \div 5$
$45 + 36$	9^2
$40 + 41$	3^4
$85 - 4$	$8^2 + 17$
$100 - 19$	$10^2 - 19$

Give 10 different expressions for 16. (Some of the expressions should include powers.)

See your learning facilitator to check your answers and to receive further instructions.

RECOGNIZING INTEGERS

What Lies Ahead

In this section you will learn these skills.

- recognizing an integer
- comparing and ordering integers

In this section you will use these words.

- positive number
- negative number
- integer
- opposite numbers

Working Together

In this section you will be learning the meaning of an integer.

Numbers greater than zero are called **positive numbers**.

Example: 1, 2, 3, 4, ...

Positive numbers can also be shown with positive signs.

Example: +1, +2, +3, +4, ...

Numbers less than zero are called **negative numbers**.

Example: −1, −2, −3, −4, ...

Whole numbers with positive signs or negative signs are called **integers**.

Example: −4, −3, −2, −1, 0, 1, 2, 3, 4, ...

Integers are used in everyday life.

Example

The levels of a parkade can be represented by integers.

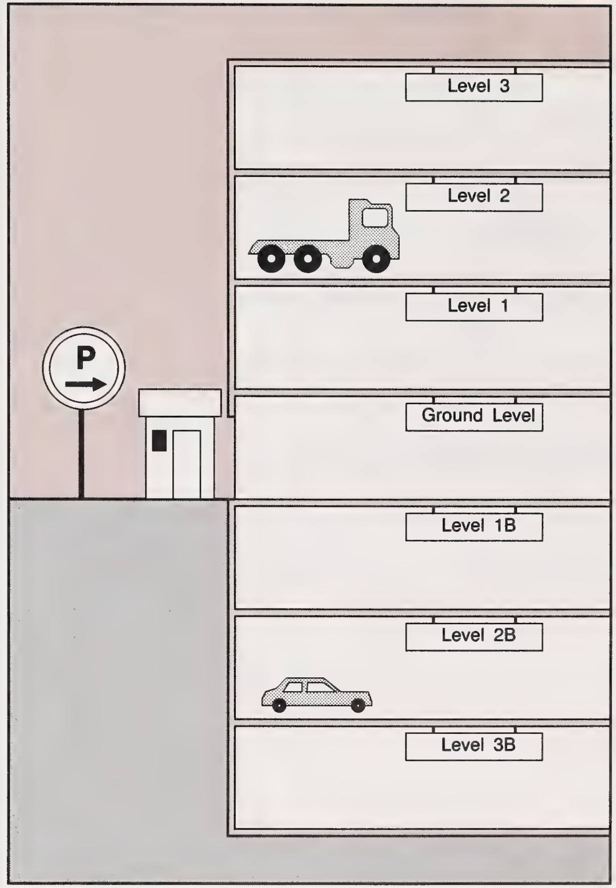
The ground level could be shown as 0.

Levels above ground level could be shown as positive integers such as $+1$, $+2$, $+3$ or 1, 2, 3.

Levels below ground level could be shown as negative integers such as -1, -2, -3.

The truck is two levels above ground level. This can be shown as $+2$ or 2.

The car is two levels below ground level. This can be shown as -2.

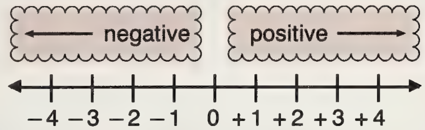


Note

- $+2$ or 2 and -2 are opposite numbers. They are the same distance from the starting point or ground level but in opposite directions.

Integers can be located on a number line.

A number line may be horizontal.



Note

The numbers on a horizontal line increase as you move from left to right and decrease as you move from right to left.

$+4$ is to the right of $+1$, making $+4 > +1$.

$+1$ is to the right of -2, making $+1 > -2$.

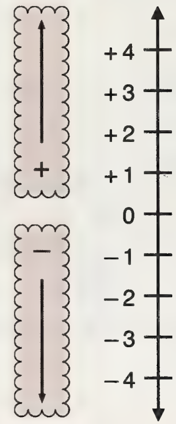
-1 is to the right of -4, making $-1 > -4$.

-4 is to the left of -1, making $-4 < -1$.

-2 is to the left of $+1$, making $-2 < +1$.

$+1$ is to the left of $+4$, making $+1 < +4$.

A number line can be vertical.



Note

The numbers on vertical number lines increase as you move upward and decrease as you move downward.

$+4$ is above $+3$, making $+4 > +3$.

-1 is above -4, making $-1 > -4$.

$+1$ is below $+4$, making $+1 < +4$.

-4 is below -3, making $-4 < -3$.

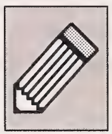
Practice Activities

Space for Your Work

Computer Alternative

1. Do Lessons 11 and 12 of the "Pre-Algebra" disk in the SRA *Computer and Instruction: Mathematics*, Level D package. Read the instructions in the folder with the disk. Remember to hold down the SHIFT key and press the ? key when you need help or have a question.

Print Alternative



2. Write the positive or negative number for each of the following situations.

   a. 5°C below freezing

   b. 1 floor below ground level

   c. 30 m above sea level

   d. 3 extra points

   e. 6 steps in front of

   f. 50 m below sea level

   g. 6 paces behind

   h. 1 under par

Space for Your Work

3. In a game, Dennis scored 28 points and Eileen scored 15. Jim lost 25 points and Theresa lost 13. Represent the scores with positive and negative numbers.

4. Kathy uses positive numbers to represent money she receives each week. She uses negative numbers for money she needs to spend. Write the number Kathy uses for each of these cases.

   a. babysitting: $10

   b. entertainment: $4

   c. snacks: $3

   d. allowance: $5

   e. music tape: $8

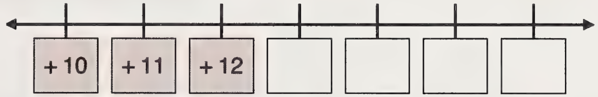
5. Name the pair of opposites that are included in each of these lists.

   a. $+400$, $+40$, -4, $+4$

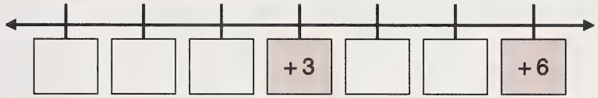
   b. $+17$, $+7$, -17, $+71$

6. Complete each number line by putting an integer in each box.

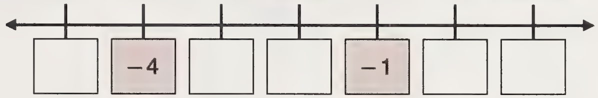
a.



b.



c.



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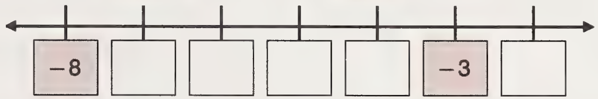

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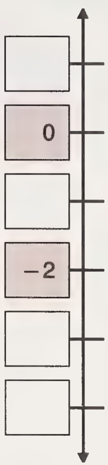

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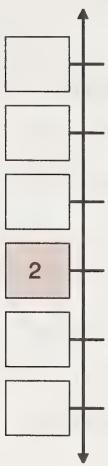

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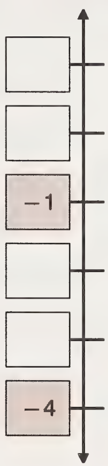

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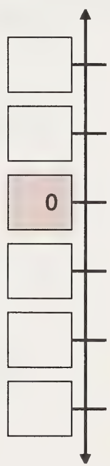
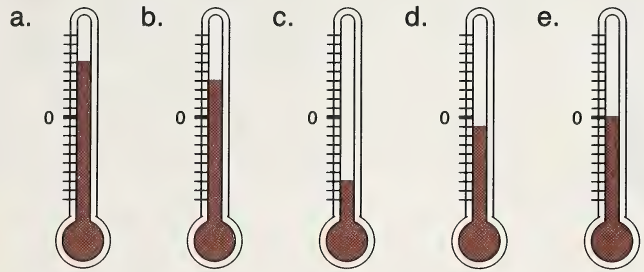


Space for Your Work

7. A thermometer is really a vertical number line. What is the temperature in degrees Celsius on each of these thermometers?



8. Use > or < to make a true statement for each of these.

a. +6 ◯ +1

b. −5 ◯ +2

c. −6 ◯ −2

d. +1 ◯ −2

e. −2 ◯ −6

f. −2 ◯ +8

Space for Your Work

Space for Your Work

9. Arrange each group of integers from least to greatest.

   a. $+2, -1, 0, +7, -5$

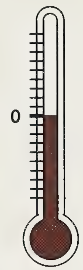
   b. $-7, +1, -4, +3, +2$

See your learning facilitator to check your answers and to receive further instructions.

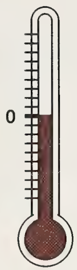
Concluding Activities

Space for Your Work

1. Compare the thermometers below with the thermometer directly to the right. Did the temperature increase or decrease in each case? How much?



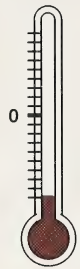
a.



b.

c.

d.



e.

Space for Your Work

2. **Longitude** is distance, in degrees, east or west on the Earth's surface.

   Earth rotates once or goes through 360° every 24 hours. Therefore each 15° zone around the globe has a difference of 1 hour in time.

   Greenwich, England, has longitude 0°. The time for places in the zone 30° west of Greenwich is 2 hours earlier. The time for places in the zone 45° east of Greenwich is 3 hours later.

   The following map shows the approximate location of the 24 time zones. The numbers indicate clock changes earlier or later than Greenwich time.

   Give the time change for each distance as a positive or negative number.

   a. London to Sydney

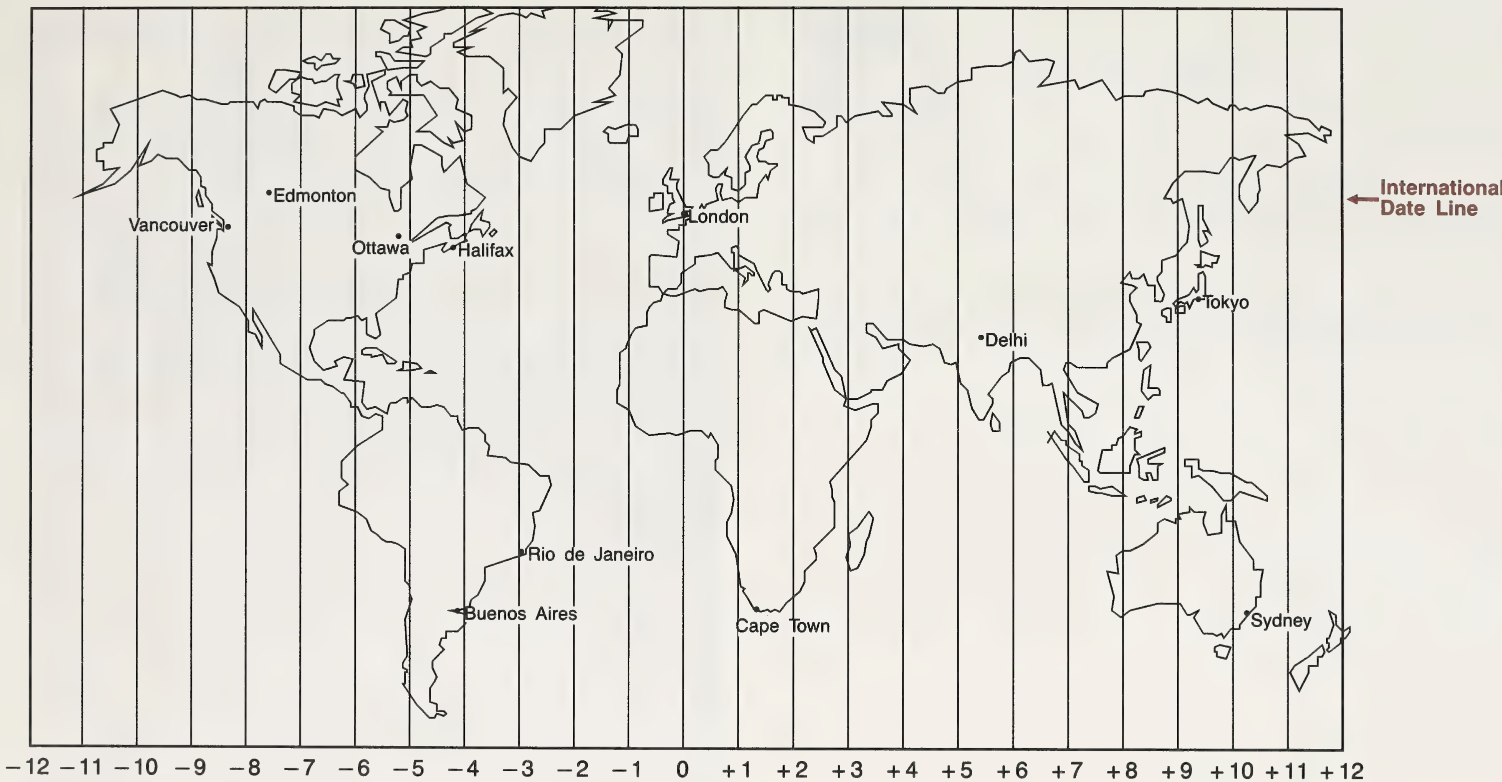
   b. London to Halifax

   c. New Delhi to Tokyo

   d. Tokyo to Rio de Janeiro

   e. Edmonton to Cape Town

   f. Halifax to Buenos Aires



See your learning facilitator to check your answers and to receive further instructions.

ADDING INTEGERS

What Lies Ahead

In this section you will learn these skills.

- adding integers using counters
- adding integers using number lines

Working Together

In the previous section you learned to read, write, and order integers and to relate integers to everyday events.

In this section you learn how to add integers both using learning aids and using number lines.

Using Counters to Model Integers

To model integers with counters you can think of a counter as either having a positive electrical charge and a negative electrical charge.

Example: How do you model $+3$?

⊕ ⊕ ⊕

Example: How do you model -4?

⊖ ⊖ ⊖ ⊖

Since a positive charge and a negative charge are opposites, combining a positive counter and a negative counter will neutralize the charges and give no charge or zero.

How do you model 0 or zero?

Example

Zero is modelled by a positive counter and a negative counter used together.

⊕ ⊖

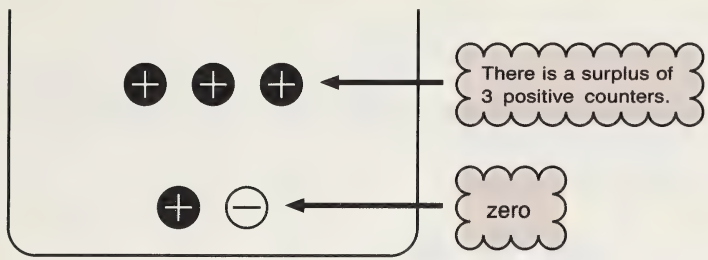
Since the positive and negative charges are balanced, the value is zero.

You can use any number of pairs to represent 0, as long as the positive and negative are balanced.

Example:

⊕ ⊖ ⊕ ⊖ ⊕ ⊖ ⊕ ⊖

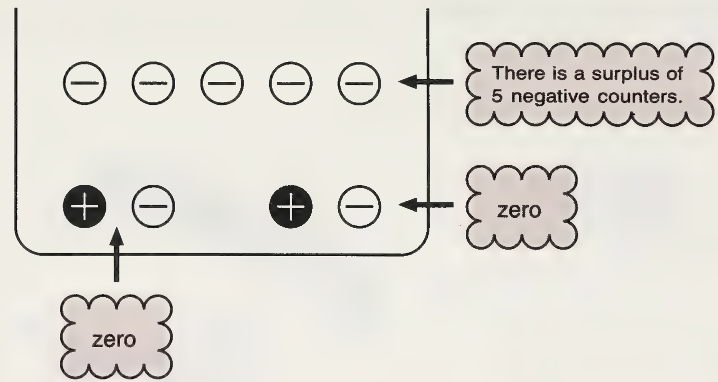
What is the charge in this container?



A positive counter and a negative counter neutralize each other. There is a surplus of 3 positive counters.

So the charge is +3.

What is the charge in this container?



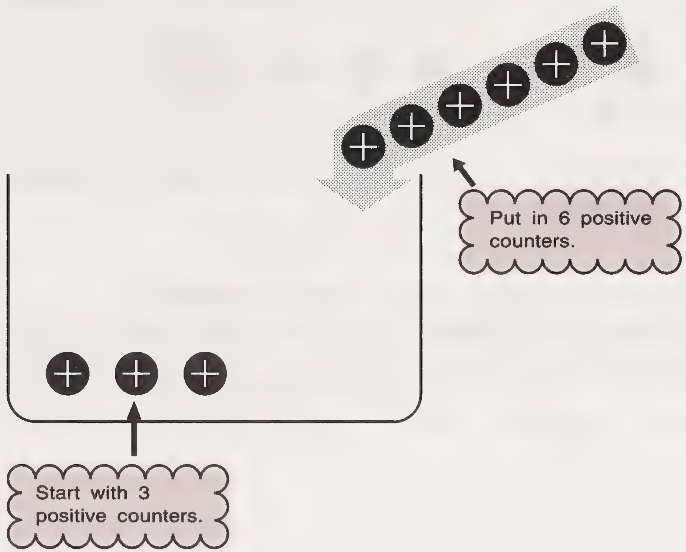
2 positive counters and 2 negative counters neutralize each other. There is a surplus of 5 negative counters.

So the charge is −5.

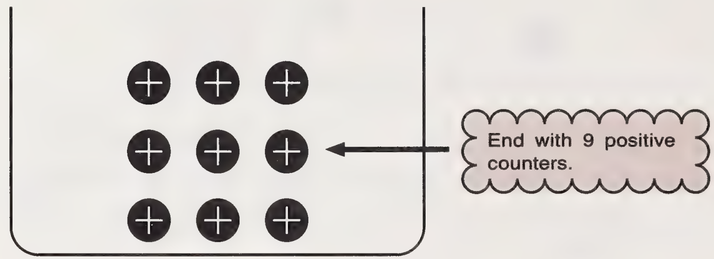
Adding Integers Using Counters

You can think of adding integers as putting counters into a container.

Example 1: $(+3) + (+6)$

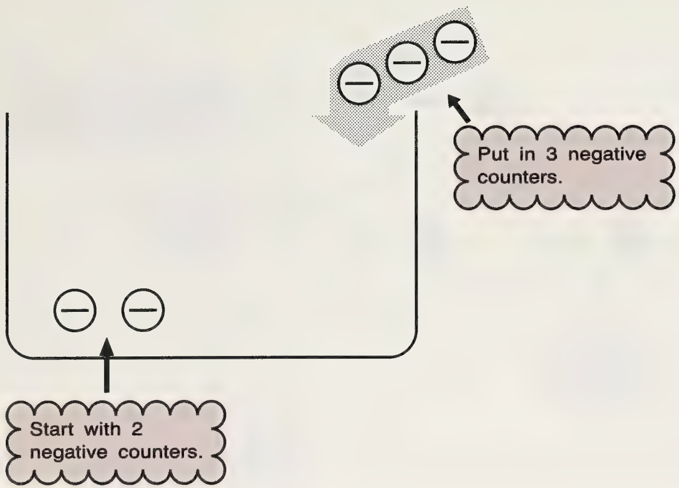


The result is 9 positive counters in the container.

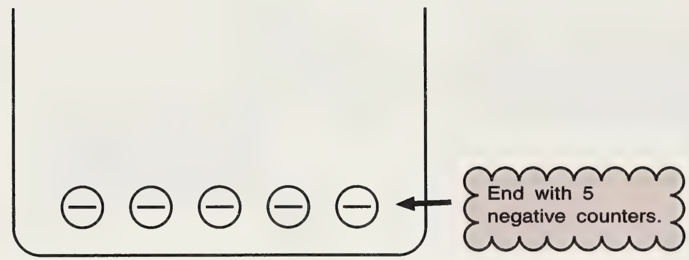


So $(+3) + (+6) = +9$.

Example 2: $(-2) + (-3)$

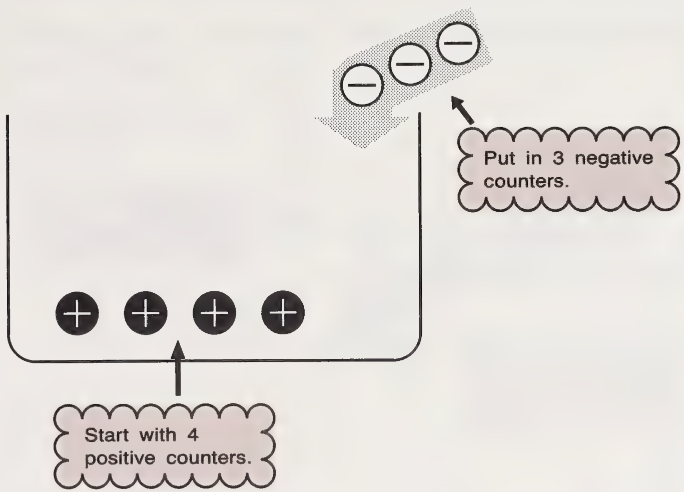


The result is 5 negative counters in the container.

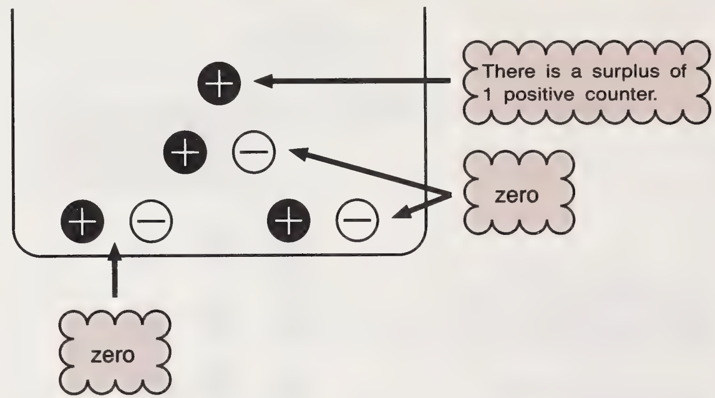


So $(-2) + (-3) = -5$.

Example 3: $(+4) + (-3)$

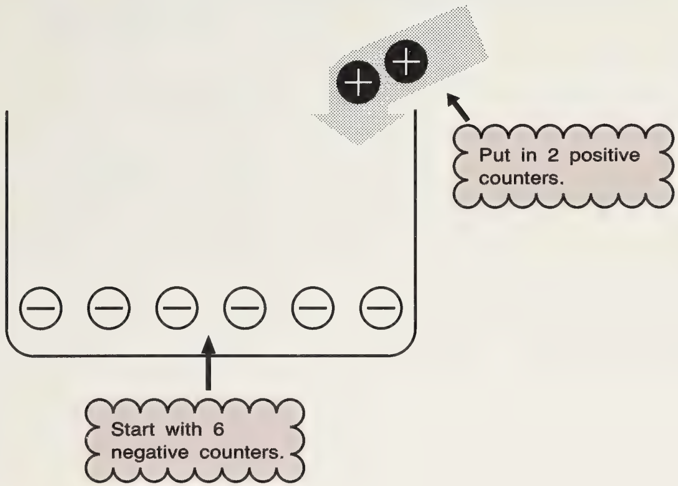


The result is a surplus of 1 positive counter.

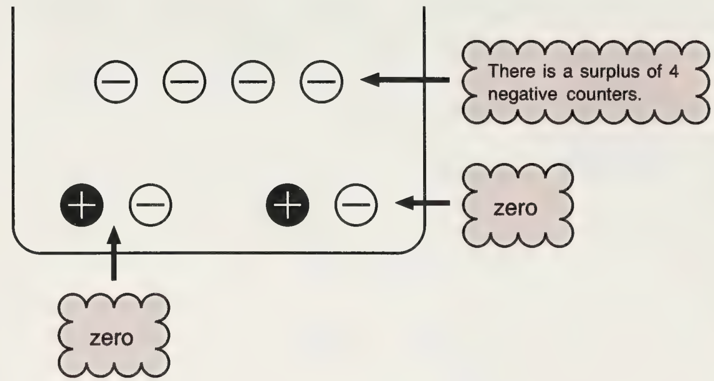


So $(+4) + (-3) = +1$.

Example 4: $(-6) + (+2)$



The result is a surplus of 4 negative counters.



So $(-6) + (+2) = -4.$

Introductory Activities

Space for Your Work

1. Use the counters at the end of this section (or use checkers or two-coloured bingo chips) to model these sums.

   a. $(+2) + (+5)$

   b. $(-3) + (-6)$

   c. $(-3) + (+6)$

   d. $(+3) + (-4)$

   e. $(-2) + (+1)$

2. a. What pattern occurs when you add counters with like signs?

   b. What pattern occurs when you add counters with unlike signs?

Space for Your Work

3. Use the patterns in questions 2 to predict the following sums.

   a. $(+7) + (+1)$

   b. $(-3) + (-6)$

   c. $(-4) + (+9)$

   d. $(+2) + (-2)$

   e. $(-6 + (+3)$

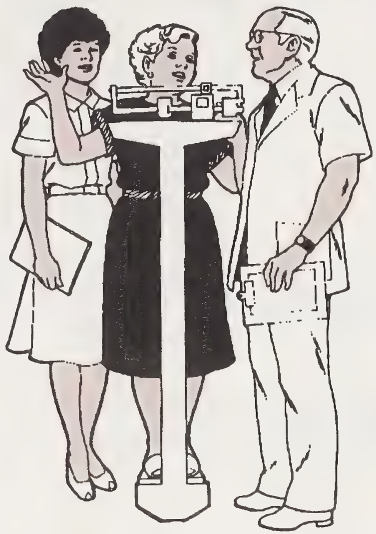
See your learning facilitator to check your answers and to receive further instructions.

Working Together

The following examples present everyday events in which integers might be added.

Number lines are included to help you picture the events and follow the calculations.

Use your counters to reinforce the calculations if you wish.



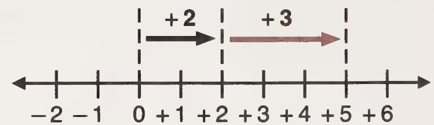
Example 1

Susan gained 2 kg in weight. Later she gained another 3 kg. How much did she gain altogether?

These events can be described by the following number sentence:

$$(+2) + (+3) = \square$$

The events can also be "pictured" on a number line.



The black arrow represents the first gain. The coloured arrow represents the second gain. The result is a total gain of 5 kg since the movement is 5 spaces to the right.

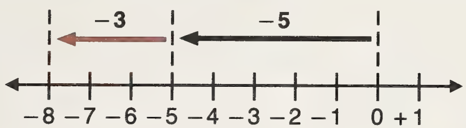
Example 2

A temperature drop of 5°C was followed by another drop of 3°C. What was the total change in temperature?

The following number sentence describes these events.

$$(-5) + (-3) = \square$$

The number line below pictures these events.



The black arrow represents the first temperature drop and the coloured arrow represents the second drop. The total movement is 8 spaces to the left. So the result is a total drop of 8°C.

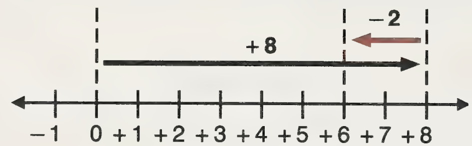
Example 3

A football team gained 8 yards on one play and then lost 2 yards on the next play. How many yards were gained or lost on the two plays together?

These events are described by the following number sentence:

$$(+8) + (-2) = \square$$

and by the following number line:



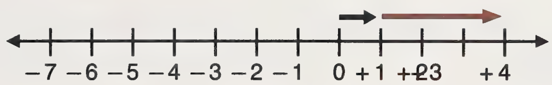
The black arrow represents the gain on the first play, while the coloured arrow represents the loss on the second play. The movement is 8 spaces to the right followed by a movement of 2 spaces to the left. The result is a net gain of 6 yards.

Practice Activities

Space for Your Work

1. Write the addition sentence suggested by the arrow on the number line.

a.



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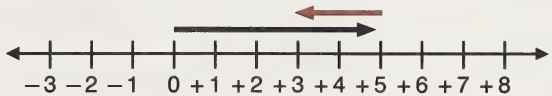
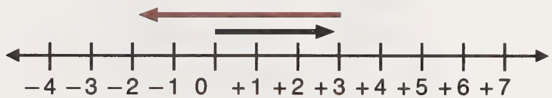
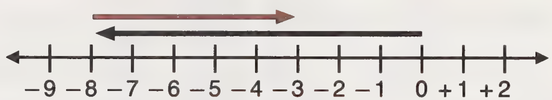


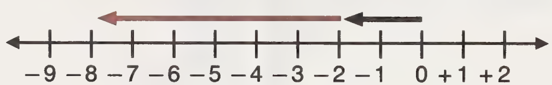
c.



d.



e.



Space for Your Work

2. Use arrows on a number line to show these sums.

   a. $(+5) + (+2)$

   b. $(-2) + (-3)$

   c. $(-1) + (-1)$

   d. $(-6) + (+5)$

   e. $(+4) + (-4)$

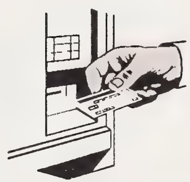
Space for Your Work

3. Draw a number line to show these events.

   a. a growth of 2 cm followed by a growth of 5 cm

   b. a temperature rise of 8°C followed by a drop of 5°C

   c. a withdrawal of $5 followed by a deposit of $10



4. Write a number sentence describing each of the events in Question 3 above.

Space for Your Work

5. Solve. Show a number sentence for each problem.

   a. The temperature at 3 o'clock was 12°C. By 9 o'clock it had fallen 5°C. What was the temperature at 9 o'clock?

   b. Colonel Bogey scores -1 on his first hole of golf and $+4$ on the next. What is his score after the two holes have been played?

   c. Felina lost 6 marbles in one game and then lost 10 more in another game. What was her total loss?

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

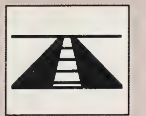
Complete the following Magic Square.

Remember that in a magic square every row, column, and diagonal must add to the same number.

−4		
	5	
8		14

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY



What Lies Ahead

In this summary you will review the skills you learned in Sections 21-29:

- identifying multiples and factors of whole numbers
- identifying prime factors, prime numbrs, and composite numbers
- using divisibility rules
- recognizing numbers written as powers
- writing the value of a power
- writing standard numbers as powers
- recognizing an integer
- comparing and ordering integers
- adding integers using counters
- adding integers using number lines

Working Together

At this point, it is a good idea to review the skills you have learned in Sections 21-29.

Turn to Section 21 and review the Pretest. Correct any errors you may have made. You may be pleasantly surprised to discover how much you have learned.

MODULE CONCLUSION

What Lies Ahead

The assignment in this Module Conclusion will evaluate the achievements of the objectives for this module.

Working Together

Now that you have done the required practice, you should do the assignment for Module 2.

Module Assignment

Turn to the Assignment Booklet and complete the assignment independently. You may refer to your notes, but do not get help from anyone.

Afterwards, submit the assignment for a grade and feedback.

APPENDIX

GLOSSARY

Addends: numbers which are added

$$\underbrace{2 + 3}_{\text{addends}} = 5$$

Additive identity: zero (When zero is added to or subtracted from a number the result is the number itself.)

$8 + 0 = 8 \qquad 7 - 0 = 7$

Associative property of addition: When there are three or more addends, the numbers can be grouped and added in any order.

$(5 + 3) + 8 = 5 + (3 + 8)$

Associative property of multiplication: when there are three or more factors, the numbers can be multiplied in any order

$(5 \times 2) \times 3 = 5 \times (2 \times 3)$

Base (of a power): the number being multiplied by itself

$2^5 \longleftarrow$ base

Common factor: a number that is a factor of two or more numbers

3 is a common factor of 18 and 15.

Common multiple: a number that is a multiple of two or more numbers

18 is a common multiple of 2 and 3.

Commutative property of addition: When there are two addends, the numbers can be added in either order.

$5 + 3 = 3 + 5$

Commutative property of multiplication: When there are two factors, the numbers can be multiplied in either order.

$5 \times 2 = 2 \times 5$

Composite number: a whole number with two or more factors

6 is a composite number (the factors are 1, 2, and 3)

Difference: the number which results when one number is subtracted from another

$$7 - 2 = \underset{\text{difference}}{\underset{\uparrow}{5}}$$

Digit: any of the individual symbols used to write numerals

The digits of 198 are 1, 9, and 8.

Distributive property: A product can be written as the sum or difference between two products.

$$5 \times (40 + 8) = 5 \times 40 + 5 \times 8$$

$$6 \times (100 - 2) = 6 \times 100 - 6 \times 2$$

Dividend: the number which is being divided

$$\underset{\substack{\uparrow \\ \text{dividend}}}{8} \div 2 = 4 \quad \text{or} \quad 2\overset{4}{\overline{\smash{)}8}} \leftarrow \text{dividend}$$

Divisible: A number is divisible by another number if the quotient has a remainder of zero.

$$\begin{array}{r} 6 \\ 3\overline{\smash{)}18} \\ \underline{18} \\ 0 \end{array} \leftarrow \text{remainder} \qquad \begin{array}{r} 6 \\ 3\overline{\smash{)}19} \\ \underline{18} \\ 1 \end{array} \leftarrow \text{remainder}$$

18 is divisible by 3; 19 is not divisible by 3.

Divisor: the number by which the dividend is divided

$$8 \div \underset{\substack{\uparrow \\ \text{divisor}}}{2} = 4 \quad \text{or} \quad \underset{\substack{\uparrow \\ \text{divisor}}}{2}\overset{4}{\overline{\smash{)}8}}$$

Estimation: an approximation that results from using strategies such as front-end digits, rounding, betweenness, etc.

Using front-end digits

$$\begin{array}{rcr} 98 & \rightarrow & 90 \\ \underline{+\ 72} & \rightarrow & \underline{+\ 70} \\ & & 160 \end{array} \leftarrow \text{estimation}$$

Using rounding

$$\begin{array}{rcr} 98 & \rightarrow & 100 \\ \underline{+\ 72} & \rightarrow & \underline{+\ 70} \\ & & 170 \end{array} \leftarrow \text{estimation}$$

Using betweenness

$$\begin{array}{rcr} 98 & \rightarrow & 90 \\ \underline{+\ 72} & \rightarrow & \underline{+\ 70} \\ & & 160 \\ & & \uparrow \\ & & \text{estimation} \end{array} \qquad \begin{array}{rcr} 98 & \rightarrow & 100 \\ \underline{+\ 72} & \rightarrow & \underline{+\ 80} \\ & & 180 \\ & & \uparrow \\ & & \text{estimation} \end{array}$$

(The sum is between 160 and 180.)

Evaluate: to find the value of

$$\begin{aligned} 7 + 2 \times 3 &= 7 + 6 \\ &= 13 \end{aligned}$$

Expanded form: The writing of a numeral as the sum of the product of each digit in the numeral and its place value

$782 = (7 \times 100) + (8 \times 10) + (2 \times 1)$

Exponent (of a power): the number which shows how many times the base is used as a factor

$2^5 \longleftarrow$ exponent

Factor (noun): in multiplication the numbers being multiplied

$\underbrace{3 \times 2}_{\text{factors}} = 6$

Factor (verb): expressing a whole number as a product of its factors

$63 = 7 \times 9$

Factor tree: a diagram used to find the prime factors of a number

63

7 × 9

7 × 3 × 3

$63 = 3 \times 3 \times 7$

Greatest common factor (GCF): the largest number that is a factor of two or more numbers

4 is the GCF of 12, 16, and 24.

Multiple of a number: the product of a number and a whole number

The multiples of 2 are $\underset{2 \times 0}{\underset{\uparrow}{0}}$, $\underset{2 \times 1}{\underset{\uparrow}{2}}$, $\underset{2 \times 2}{\underset{\uparrow}{4}}$, $\underset{2 \times 3}{\underset{\uparrow}{6}}$, $\underset{2 \times 4}{\underset{\uparrow}{8}}$, ...

Multiplicative identity: one (When a number is multiplied or divided by one, the result is the number itself.)

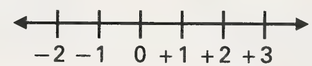
$5 \times 1 = 5$

$6 \div 1 = 6$

Negative number: any number less than 0

-2 is a negative number.

Number line: a line used to illustrate a set of numbers



Numeral: a symbol or group of symbols that represents a number

5, 12, and 352 are numerals.

Opposite numbers: negative and positive numbers that are the same distance from zero but in opposite directions

+2 and − 2 are opposite numbers.

Parentheses: the symbols ( ). When used with operation signs, parentheses help indicate the order of operations.

$6 - (2 + 3)$

2 and 3 must be combined first.

Place value: the value given to the place where a digit appears in a numeral

In 435, 3 is in the tens place.

Positive numbers: numbers greater than 0

Power: a product of equal factors

$$\begin{aligned} 81 &= 3 \times 3 \times 3 \times 3 \\ &= 3^4 \end{aligned}$$

81 is the fourth power of 3.

Prime factor: a factor that is a prime number

Prime factorization: the writing of a number as the product of its prime factors

$30 = 2 \times 3 \times 5$

Prime number: a whole number with exactly two different factors, itself and 1

2, 3, 5, 7, 9, 11, 13, 17, and 19 are prime numbers.

Product: the result of multiplication

Quotient: the result of division

Remainder: the number left when division is not exact

Standard form: the usual form of a numeral

Subtrahend: the number that is subtracted from another number

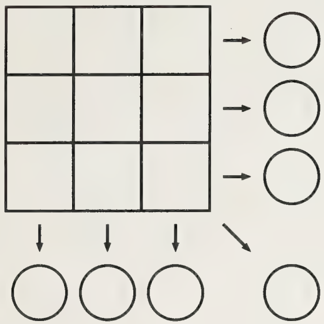
Undefined: a very larger number or a very small number which cannot be expressed mathematically

PLACE ROLL

5	3	7	PRIME
2	6	8	17
PRIME	21	4	16
13	9	PRIME	26

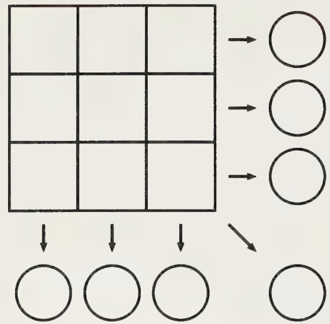
CROSS OUT SINGLES

Round 1



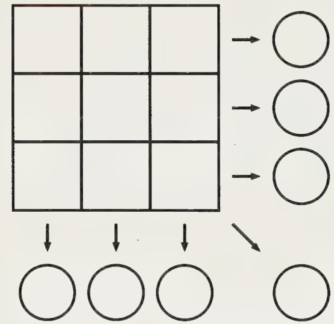
Score: ______

Round 2



Score: ______

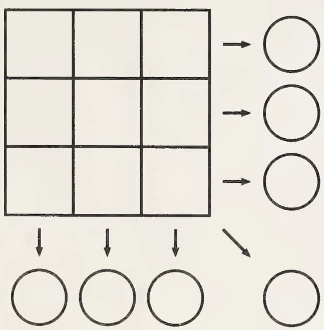
Round 3



Score: ______

Grand Total: ______

Round 1



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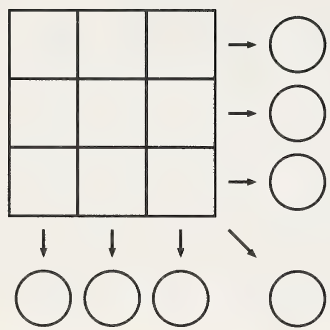
Round 2



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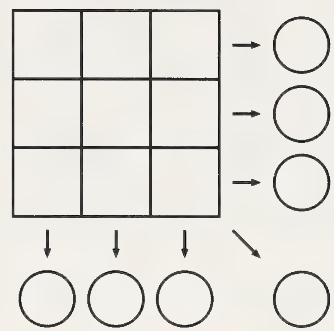
Round 3



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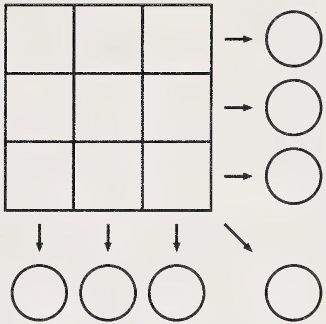
Grand Total: ______

CROSS OUT SINGLES

Round 1



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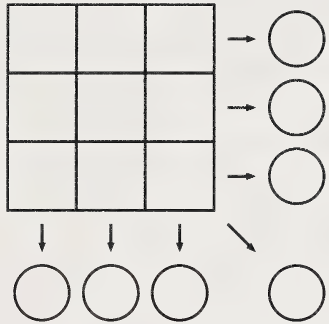

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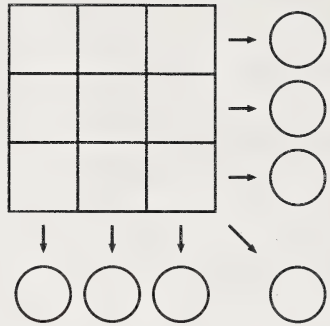
Round 3



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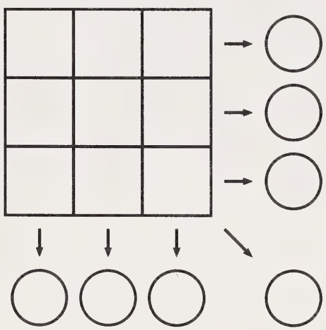
Round 1



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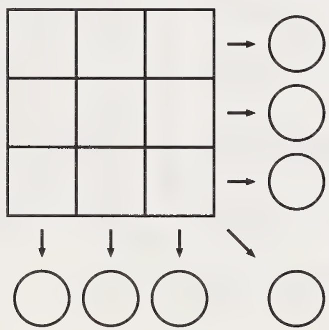

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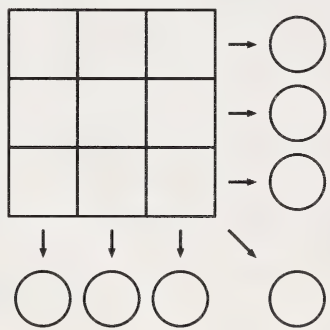

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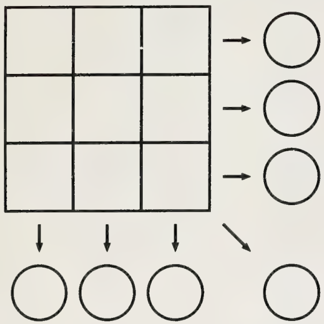
Grand Total: ______

CROSS OUT SINGLES

Round 1



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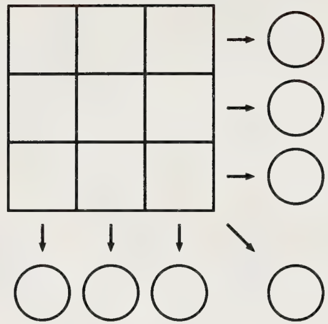

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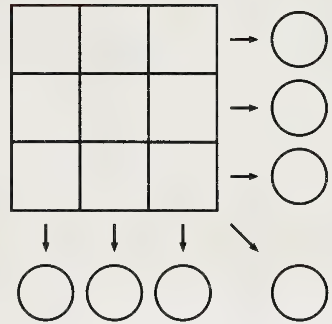

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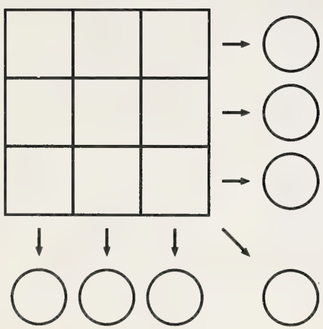
Round 1



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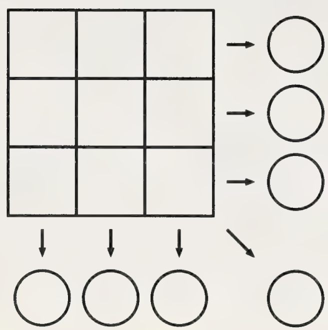

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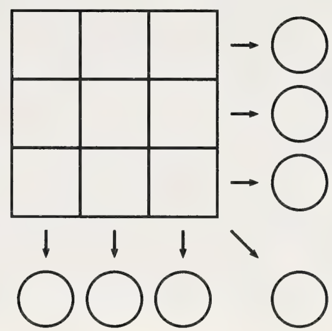

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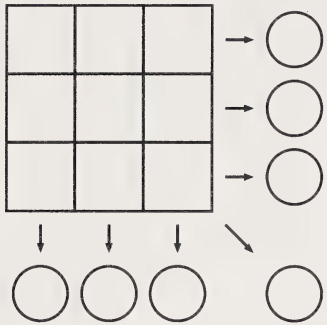
CROSS OUT SINGLES

Round 1



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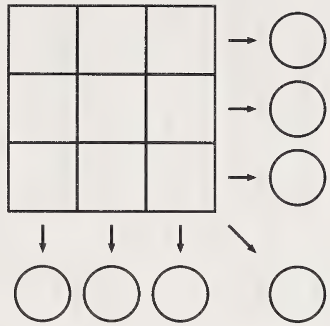
Round 2



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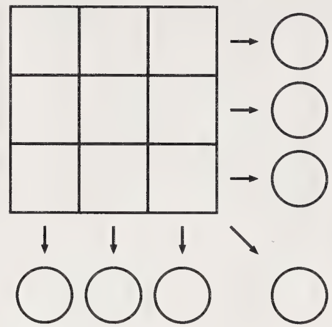

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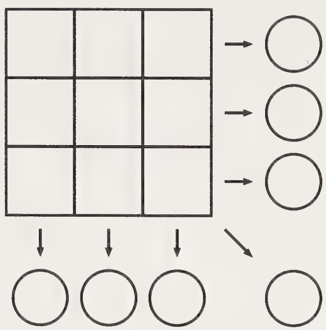
Grand Total: ____________

Round 1



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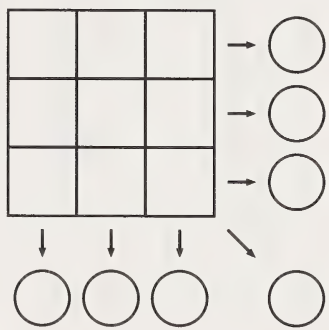

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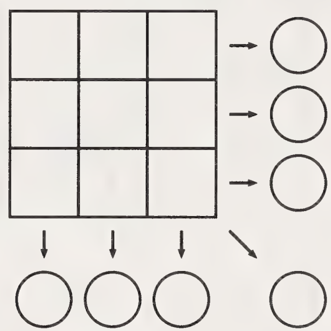


Score: ____________

Grand Total: ____________

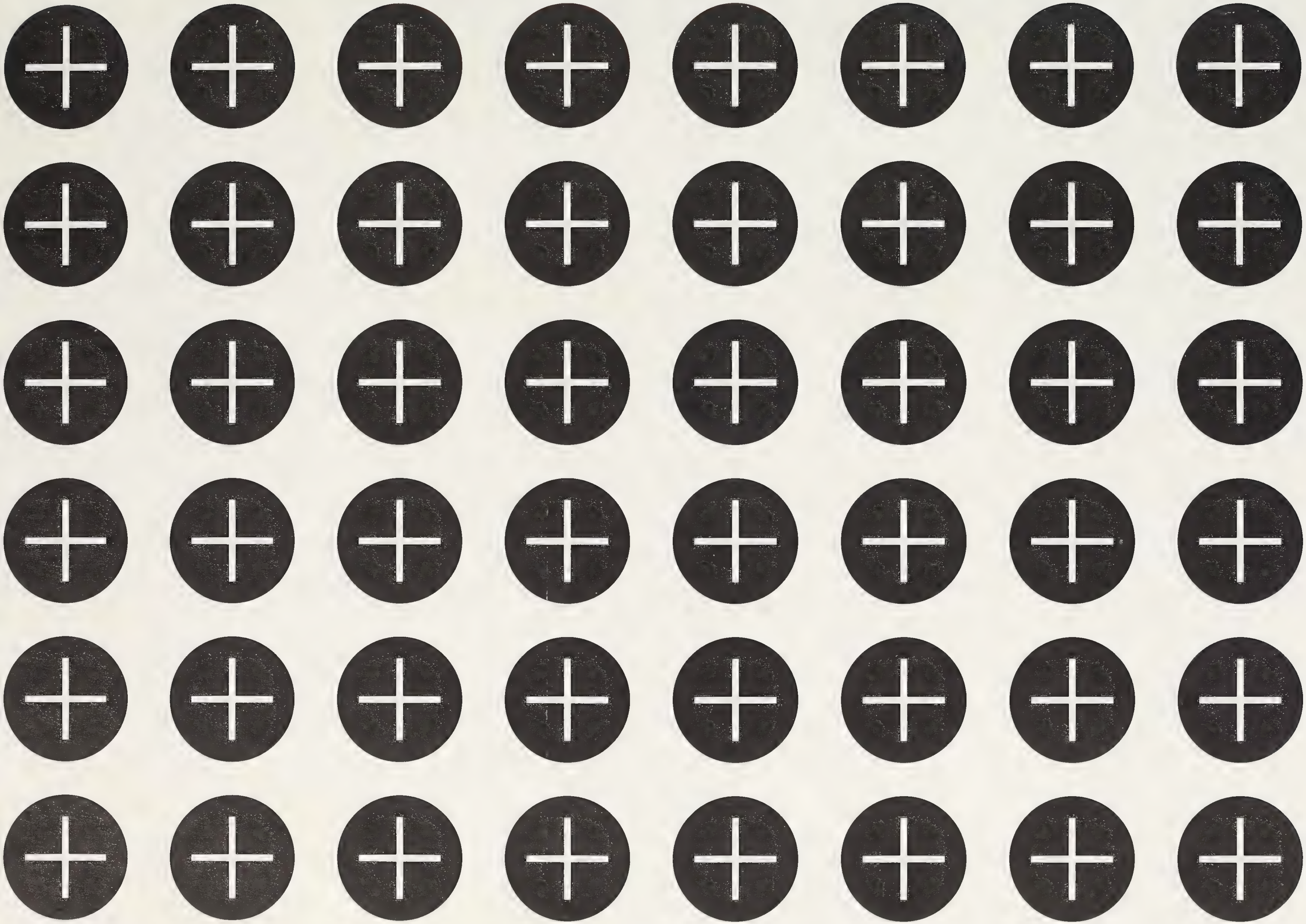
MULTIPLE TIC-TAC-TOE

14	20	6	30
9	4	15	3
1	10	5	12
8	18	24	2

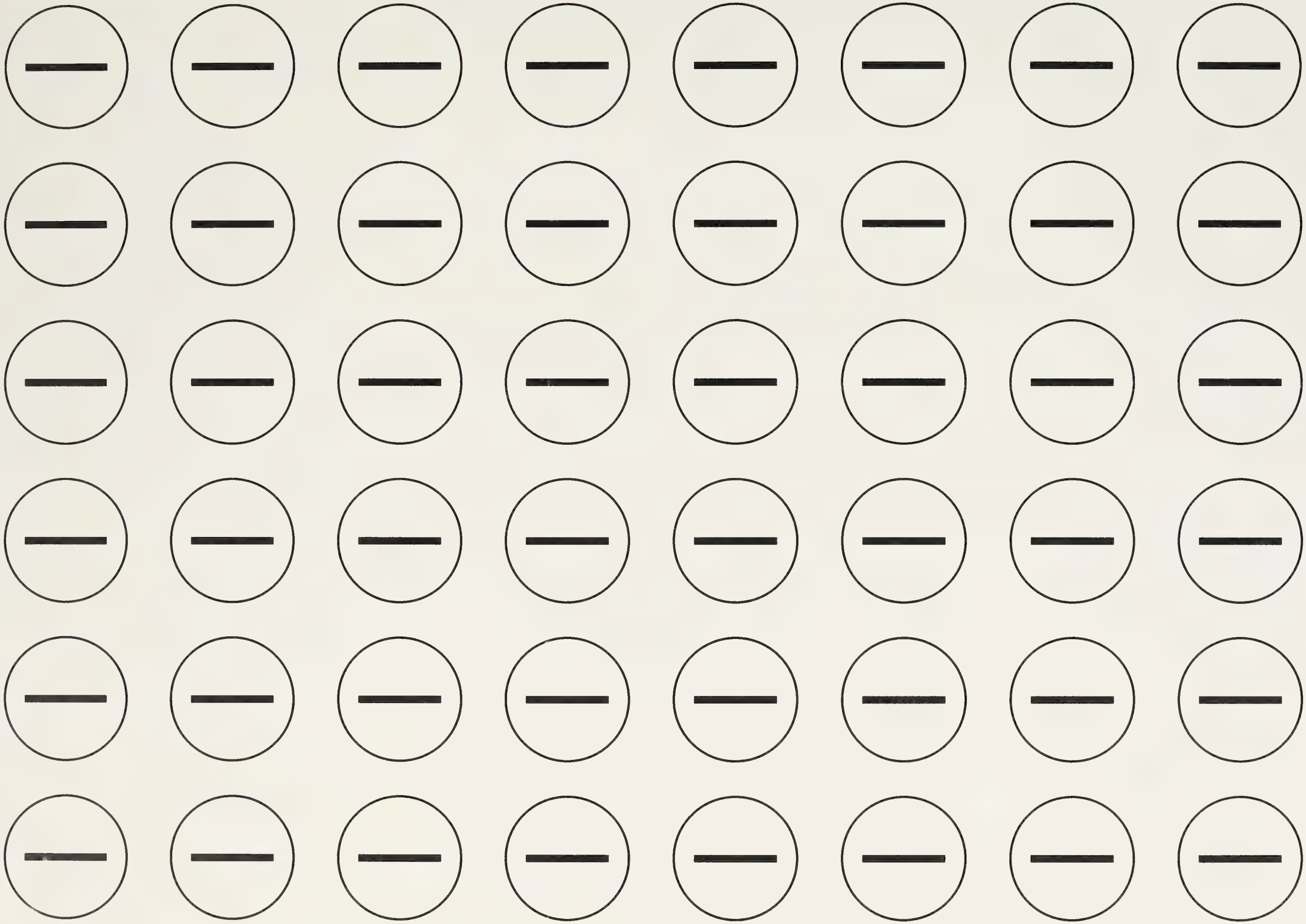
TAX COLLECTOR

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

POSITIVE COUNTERS



NEGATIVE COUNTERS



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